TABLA DE DERIVADAS E INTEGRALES

19hs
MIERCOLES
S +
I -

PREediciones

COLECCION

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INDICE

NUMEROS NOTABLES

$\pi = 3{,}14159...$ $\qquad e = 2{,}718281...$

$\sqrt{2} = 1{,}414213...$ $\qquad \sqrt{3} = 1{,}73205...$

$e^{\pi} = 23{,}14069...$ $\qquad \pi^{e} = 22{,}45915...$

$e^{e} = 15{,}15426...$ $\qquad \sqrt{e} = 1{,}64872...$

$\sqrt{\pi} = 1{,}77245... = \Gamma(\frac{1}{2})$ (Γ : función gamma)

$\Gamma(\frac{1}{3}) = 2{,}67893...$ $\qquad \Gamma(\frac{1}{4}) = 3{,}6250...$

$\gamma = 0{,}57721566...$ (constante de Euler)

$1 \text{ radián} = \frac{180^\circ}{\pi} = 57{,}29577...^\circ$

Rob: Simple cerrada C
- orientada
- simple por tramos.

$1^\circ = \frac{\pi}{180}$ radianes $= 0{,}01745...$ radianes

números de Euler $(E_k) = \frac{\pi^{2k+1}}{2^{2k+2}\,(2k)!} \cdot E_k$

números de Bernoulli $(B_k) = \frac{\pi^{2k} \cdot (2^{2k} - 1)}{2\,(2k)!} \cdot B_k$

FUNCIONES TRIGONOMETRICAS CIRCULARES

RELACIONES ENTRE FUNCIONES TRIGONOMETRICAS

1) $\operatorname{sen}^2 x + \cos^2 x = 1$

2) $\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$

3) $\operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x}$

4) $\sec x = \frac{1}{\cos x}$

5) $\operatorname{cosec} x = \frac{1}{\operatorname{sen} x}$

6) $1 + \operatorname{tg}^2 x = \sec^2 x$

7) $1 + \operatorname{cotg}^2 x = \operatorname{cosec}^2 x$

FUNCIONES DE LA SUMA O DIFERENCIA DE ANGULOS

1) $\operatorname{sen}(x \pm y) = \operatorname{sen} x \cdot \cos y \pm \operatorname{sen} y \cdot \cos x$

2) $\cos(x \pm y) = \cos x \cdot \cos y \mp \operatorname{sen} x \cdot \operatorname{sen} y$

3) $\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \cdot \operatorname{tg} y}$

FUNCIONES DEL DUPLO DEL ANGULO

1) $\operatorname{sen} 2x = 2 \cdot \operatorname{sen} x \cdot \cos x$

2) $\cos 2x = \cos^2 x - \operatorname{sen}^2 x = 1 - 2 \operatorname{sen}^2 x = 2 \cos^2 x - 1$

3) $\operatorname{tg} 2x = \dfrac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$

FUNCIONES DEL ANGULO MITAD

1) $\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\dfrac{1 - \cos x}{2}}$ 2) $\cos\left(\frac{x}{2}\right) = \pm \sqrt{\dfrac{1 + \cos x}{2}}$

3) $\operatorname{tg}\left(\frac{x}{2}\right) = \pm \sqrt{\dfrac{1 - \cos x}{1 + \cos x}} = \dfrac{\operatorname{sen} x}{1 - \cos x} = \dfrac{1 - \cos x}{\operatorname{sen} x} = \operatorname{cosec} x - \operatorname{cotg} x$

FUNCIONES POTENCIA

1) $\operatorname{sen}^2 x = \frac{1}{2}(1 - \cos 2x)$ 2) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

SUMA, DIFERENCIA Y PRODUCTO DE FUNCIONES

1) $\operatorname{sen} x + \operatorname{sen} y = 2 \cdot \operatorname{sen}\left[\dfrac{x + y}{2}\right] \cdot \cos\left[\dfrac{x - y}{2}\right]$ $\cos n\pi = (-1)^n$

2) $\operatorname{sen} x - \operatorname{sen} y = 2 \cdot \cos\left[\dfrac{x + y}{2}\right] \cdot \operatorname{sen}\left[\dfrac{x - y}{2}\right]$

3) $\cos x + \cos y = 2 \cdot \cos\left[\dfrac{x + y}{2}\right] \cdot \cos\left[\dfrac{x - y}{2}\right]$

4) $\cos x - \cos y = -2 \cdot \operatorname{sen}\left[\dfrac{x + y}{2}\right] \cdot \operatorname{sen}\left[\dfrac{x - y}{2}\right]$

5) $2 \cdot \operatorname{sen} x \cdot \operatorname{sen} y = \cos(x - y) - \cos(x + y)$

6) $2 \cdot \cos x \cdot \cos y = \cos(x - y) + \cos(x + y)$

7) $2 \cdot \operatorname{sen} x \cdot \operatorname{sen} y = \operatorname{sen}(x - y) + \operatorname{sen}(x + y)$

FUNCIONES TRIGONOMETRICAS HIPERBOLICAS

1) $\operatorname{senh} x = \dfrac{e^x - e^{-x}}{2}$ 2) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

3) $\operatorname{tgh} x = \dfrac{\operatorname{senh} x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$ 4) $\operatorname{cotgh} x = \dfrac{\cosh x}{\operatorname{senh} x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

5) $\operatorname{sech} x = \dfrac{1}{\cosh x}$ 6) $\operatorname{cosech} x = \dfrac{1}{\operatorname{senh} x}$

RELACIONES FUNDAMENTALES

1) $\cosh^2 x - \operatorname{senh}^2 x = 1$ 2) $\operatorname{tgh}^2 x + \operatorname{sech}^2 x = 1$

3) $\operatorname{cotgh}^2 x - \operatorname{cosech}^2 x = 1$

FUNCIONES DE LA SUMA Y DIFERENCIA DE ANGULOS

1) $\operatorname{senh}(x \pm y) = \operatorname{senh} x \cosh y \pm \operatorname{senh} y \cosh x$

2) $\cosh(x \pm y) = \cosh x \cosh y \pm \operatorname{senh} x \operatorname{senh} y$

3) $\operatorname{tgh}(x \pm y) = \dfrac{\operatorname{tgh} x \pm \operatorname{tgh} y}{1 \pm \operatorname{tgh} x \operatorname{tgh} y}$

FUNCIONES DEL DUPLO DEL ANGULO

1) $\operatorname{senh} 2x = 2 \cdot \operatorname{senh} x \cdot \cosh x$

2) $\cosh 2x = \cosh^2 x + \operatorname{senh}^2 x = 1 + 2 \operatorname{senh}^2 x = 2 \cosh^2 x - 1$

3) $\operatorname{tgh} 2x = \dfrac{2 \operatorname{tgh} x}{1 - \operatorname{tgh}^2 x}$

FUNCIONES DEL ANGULO MITAD

1) $\operatorname{senh}\left(\frac{x}{2}\right) = \pm \sqrt{\dfrac{\cosh x - 1}{2}}$ $\begin{cases} \text{si } x > 0, \text{ vale signo } + \\ \text{si } x < 0, \text{ vale signo } - \end{cases}$

2) $\cosh\left(\frac{x}{2}\right) = + \sqrt{\dfrac{1 + \cosh x}{2}}$

3) $\operatorname{tgh}\left(\frac{x}{2}\right) = \pm \sqrt{\dfrac{\cosh x - 1}{\cosh x + 1}} = \dfrac{\operatorname{senh} x}{1 + \cosh x} = \dfrac{\cosh x - 1}{\operatorname{senh} x}$

RELACION ENTRE LOS ARGUMENTOS HIPERBOLICOS Y LOGARITMICOS

1) $\operatorname{senh}^{-1} x = \ln\left[x + \sqrt{1 + x^2}\right]; \quad \forall x$

2) $\operatorname{tgh}^{-1} x = \frac{1}{2} \ln\left[\dfrac{1 + x}{1 - x}\right]; \quad |x| < 1$

3) $\operatorname{sech}^{-1} x = \ln\left[\dfrac{1}{x} + \sqrt{\dfrac{1}{x^2} - 1}\right]; \quad 0 < x \leq 1$

4) $\cosh^{-1} x = \ln\left[x + \sqrt{x^2 - 1}\right]; \quad x \geq 1$

5) $\operatorname{cotgh}^{-1} x = \frac{1}{2} \ln\left[\dfrac{1 + x}{x - 1}\right]; \quad |x| > 1$

6) $\operatorname{cosech}^{-1} x = \ln\left[\dfrac{1}{x} + \sqrt{\dfrac{1}{x^2} + 1}\right]; \quad x \neq 0$

TABLA DE DERIVADAS

REGLAS DE DERIVACION

- Las funciones u, v y w son derivables en x.
- k, r, a y n son constantes reales
- x es variable independiente

a) Regla de la cadena $\frac{d}{dx} y = \frac{d}{du} y \cdot \frac{d}{dx} u$

b) $\frac{d}{dx} y = \frac{1}{\frac{d}{dy} x}$

c) $\frac{d}{dx} y = \frac{\frac{d}{du} y}{\frac{d}{du} x}$

FUNCION	DERIVADA
k	$\varnothing$
x	1
kx	k
ku	$k \frac{d}{dx} u$
u^r	$r\, u^{r-1} \frac{d}{dx} u$
$u + v - w$	$\frac{d}{dx} u + \frac{d}{dx} v - \frac{d}{dx} w$
$u\, v$	$\frac{d}{dx} u \cdot v + u \cdot \frac{d}{dx} v$
$u\, v\, w$	$\frac{d}{dx} u \cdot v \cdot w + u \cdot \frac{d}{dx} v \cdot w + u \cdot v \cdot \frac{d}{dx} w$
$\frac{u}{v}$	$\frac{\frac{d}{dx} u \cdot v - u \cdot \frac{d}{dx} v}{v^2}; \quad v \neq \varnothing$
$\ln x$	$\frac{1}{x}$
$\ln u$	$\frac{1}{u} \cdot \frac{d}{dx} u$

FUNCION	DERIVADA
$\log_a u$	$\frac{1}{u \ln a} \cdot \frac{d}{dx} u \quad u > \varnothing,\ a > \varnothing,\ a \neq 1$
e^x	e^x
e^u	$e^u \cdot \frac{d}{dx} u$
a^u	$a^u \cdot \ln a \cdot \frac{d}{dx} u; \quad a > \varnothing,\ a \neq 1$
u^v	$u^v \left(\frac{d}{dx} v \cdot \ln u + \frac{v}{u} \cdot \frac{d}{dx} u \right); \quad u > \varnothing$
$\operatorname{sen} u$	$\cos u \cdot \frac{d}{dx} u$
$\cos u$	$- \operatorname{sen} u \cdot \frac{d}{dx} u$
$\operatorname{tg} u$	$\sec^2 u \cdot \frac{d}{dx} u$
$\operatorname{cotg} u$	$- \operatorname{cosec}^2 u \cdot \frac{d}{dx} u$
$\sec u$	$\sec u \cdot \operatorname{tg} u \cdot \frac{d}{dx} u$
$\operatorname{cosec} u$	$- \operatorname{cosec} u \cdot \operatorname{cotg} u \cdot \frac{d}{dx} u$
$\operatorname{senh} u$	$\cosh u \cdot \frac{d}{dx} u$
$\cosh u$	$\operatorname{senh} u \cdot \frac{d}{dx} u$
$\operatorname{tgh} u$	$\operatorname{sech}^2 u \cdot \frac{d}{dx} u$
$\operatorname{cotgh} u$	$- \operatorname{cosech}^2 u \cdot \frac{d}{dx} u$
$\operatorname{sech} u$	$- \operatorname{sech} u \cdot \operatorname{tgh} u \cdot \frac{d}{dx} u$
$\operatorname{cosech} u$	$- \operatorname{cosech} u \cdot \operatorname{cotgh} u \cdot \frac{d}{dx} u$

FUNCION	DERIVADA
$sen^{-1} u$ (arcsen u)	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$cos^{-1} u$ (arccos u)	$- \frac{1}{\sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$tg^{-1} u$ (arctg u)	$\frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$cotg^{-1} u$ (arccotg u)	$- \frac{1}{1+u^2} \cdot \frac{d}{dx} u$
$sec^{-1} u$ (arcsec u)	$\frac{1}{u \cdot \sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$cosec^{-1} u$ (arccosec u)	$- \frac{1}{u \sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$senh^{-1} u$ (arcsenh u)	$\frac{1}{\sqrt{u^2+1}} \cdot \frac{d}{dx} u$
$cosh^{-1} u$ (arccosh u)	$\frac{1}{\sqrt{u^2-1}} \cdot \frac{d}{dx} u$
$tgh^{-1} u$ (arctgh u)	$\frac{1}{1-u^2} \cdot \frac{d}{dx} u$
$cotgh^{-1} u$ (arccotgh u)	$\frac{1}{1-u^2} \cdot \frac{d}{dx} u$
$sech^{-1} u$ (arcsech u)	$- \frac{1}{u \sqrt{1-u^2}} \cdot \frac{d}{dx} u$
$cosech^{-1} u$ ($arcosech^{-1}$ u)	$- \frac{1}{u \sqrt{1+u^2}} \cdot \frac{d}{dx} u$

TABLA DE INTEGRALES

INTEGRALES INDEFINIDAS

REGLAS PARA UNA INTEGRACION

* Las f, u, v y w son funciones de x.
* a, b, p, q, r y n son constantes, r es real y n es natural.

1 $\int a\, dx = ax$

2 $\int a f(x)\, dx = a \int f(x)\, dx$

3 $\int (u \pm v \pm w \pm \cdots)\, dx = \int u\, dx \pm \int v\, dx \pm \int w\, dx \pm \cdots$

4 $\int u\, dv = uv - \int v\, du$ Integración por partes

5 $\int f(ax)\, dx = \frac{1}{a} \int f(u)\, du$ Cambio de variable $u = ax$

6 $\int F\{f(x)\}\, dx = \int F(u) \frac{dx}{du}\, du = \int \frac{F(u)}{f'(x)}\, du$

7 $\int x^r\, dx = \frac{x^{r+1}}{r+1}$; Con $r \neq -1$. Para $r = -1$ ver 8

8 $\int \frac{1}{x}\, dx = \ln|x| = \begin{cases} \ln x \text{ si } x > 0 \\ \ln(-x) \text{ si } x < 0 \end{cases}$; $x \neq 0$

9 $\int e^x\, dx = e^x$

10 $\int a^x\, dx = \frac{a^x}{\ln a} = a^x \log_a e$ Para $a > 0$ y $a \neq 1$

11 $\int sen\, x\, dx = -\cos x$

12 $\int \cos x\, dx = sen\, x$

13 $\int tg\, x\, dx = \ln \sec x = -\ln \cos x$

14 $\int cotg\, x\, dx = \ln sen\, x$

15 $\int \sec x\, dx = \ln(\sec x + tg\, x) = \ln tg\left(\frac{x}{2} + \frac{\pi}{2}\right)$

16 $\int cosec\, x\, dx = \ln(cosec\, x - cotg\, x) = \ln tg \frac{x}{2}$

17 $\int \sec^2 x \, dx = tg\, x$

18 $\int cosec^2 x \, dx = -\, cotg\, x$

19 $\int tg^2 x \, dx = tg\, x - x$

20 $\int cotg^2 x \, dx = -\, cotg\, x - x$

21 $\int sen^2 x \, dx = \frac{x}{2} - \frac{sen\, 2x}{4} = \frac{1}{2} (x - sen\, x \cos x)$

22 $\int \cos^2 x \, dx = \frac{x}{2} + \frac{sen\, 2x}{4} = \frac{1}{2} (x + sen\, x \cos x)$

23 $\int \sec x \, tg\, x \, dx = \sec x$

24 $\int cosec\, x \, cotg\, x \, dx = -\, cosec\, x$

25 $\int senh\, x \, dx = \cosh x$

26 $\int \cosh x \, dx = senh\, x$

27 $\int tgh\, x \, dx = \ln \cosh x$

28 $\int cotgh\, x \, dx = \ln senh\, x$

29 $\int sech\, x \, dx = sen^{-1} x\, (tgh\, x)$ ó $2\, tg^{-1} e^x$

30 $\int cosech\, x \, dx = \ln tgh \frac{x}{2}$ ó $-\, cotgh^{-1} e^x$

31 $\int sech^2 x \, dx = tgh\, x$

32 $\int cosech^2 x \, dx = -\, cotgh\, x$

33 $\int tgh^2 x \, dx = x - tgh\, x$

34 $\int cotgh^2 x \, dx = x - cotgh\, x$

35 $\int senh^2 x \, dx = \frac{senh\, 2x}{4} - \frac{x}{2} = \frac{1}{2} (senh\, x \cosh x - x)$

36 $\int \cosh^2 x \, dx = \frac{senh\, 2x}{4} + \frac{x}{2} = \frac{1}{2} (senh\, x \cosh x + x)$

37 $\int sech\, x \, tgh\, x \, dx = -\, sech\, x$

38 $\int cosech\, x \, cotgh\, x \, dx = -\, cosech\, x$

39 $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} tg^{-1} \frac{x}{a}$

40 $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left(\frac{x - a}{x + a} \right) = -\frac{1}{a} cotgh^{-1} \frac{x}{a} \qquad x^2 > a^2$

41 $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left(\frac{x + a}{a - x} \right) = \frac{1}{a} tgh^{-1} \frac{x}{a} \qquad x^2 < a^2$

42 $\int \frac{dx}{\sqrt{a^2 - x^2}} = sen^{-1} \frac{x}{a}$

43 $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left(x + \sqrt{a^2 + x^2} \right)$ ó $senh^{-1} \frac{x}{a}$

44 $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left(x + \sqrt{x^2 - a^2} \right)$ ó $\cosh^{-1} \frac{x}{a}$

45 $\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right|$

46 $\int \frac{dx}{x \sqrt{x^2 + a^2}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right)$

47 $\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 - x^2}}{x} \right)$

48 $\int f^{(n)} g \, dx = f^{(n-1)} g - f^{(n-2)} g' + f^{(n-3)} g'' - \dots + (-1)^n \int f\, g^{(n)} \, dx$

METODO DE SUSTITUCION

49 $\int F(ax + b) \, dx = \frac{1}{a} \int F(u) \, du \qquad u = ax + b$

50 $\int F\left(\sqrt{ax + b} \right) dx = \frac{2}{a} \int u\, F(u) \, du \qquad u = \sqrt{ax + b}$

51 $\int F\left(\sqrt[n]{ax + b} \right) dx = \frac{n}{a} \int u^{n-1} F(u) \, du \qquad u = \sqrt[n]{ax + b}$

52 $\int F\left(\sqrt{a^2 - x^2} \right) dx = a \int F(a \cos u) \cos u \, du \qquad x = a\, sen\, u$

53 $\int F\left(\sqrt{x^2 + a^2} \right) dx = a \int F(a \sec u) \sec^2 u \, du \qquad x = a\, tg\, u$

54 $\int F\left(\sqrt{x^2 - a^2} \right) dx = a \int F(a \sec u) \sec u \, tg\, u \, du \qquad x = a \sec u$

55 $\int F(e^{ax}) \, dx = \frac{1}{a} \int \frac{F(u)}{u} \, du \qquad u = e^{ax}$

56 $\int F(\ln u) \, dx = \int F(u)\, e^u \, du \qquad u = \ln u$

57 $\displaystyle\int F\left(\operatorname{sen}^{-1}\frac{x}{a}\right)dx = a\int F(u)\cos u\,du \qquad u = \operatorname{sen}^{-1}\frac{x}{a}$

Para otras funciones trigonométricas reciprocas se obtienen similares resultados.

58 $\displaystyle\int F(\operatorname{sen} x, \cos x)\,dx = 2\int F\left(\frac{2u}{1+u^2}, \frac{1-u^2}{1+u^2}\right)\frac{du}{1+u^2} \qquad u = \operatorname{tg}\frac{x}{2}$

INTEGRALES INDEFINIDAS CLASIFICADAS POR LA FORMA

INTEGRALES CON $ax + b$

59 $\displaystyle\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$

60 $\displaystyle\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$

61 $\displaystyle\int \frac{x^2\,dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$

62 $\displaystyle\int \frac{x^3\,dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b)$

63 $\displaystyle\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$

64 $\displaystyle\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$

65 $\displaystyle\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right)$

66 $\displaystyle\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$

67 $\displaystyle\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$

68 $\displaystyle\int \frac{x^2\,dx}{(ax+b)^2} = \frac{ax+b}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b)$

69 $\displaystyle\int \frac{x^3\,dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{3b^2}{a^4}\ln(ax+b)$

70 $\displaystyle\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} + \frac{1}{b^2}\ln\left(\frac{x}{ax+b}\right)$

71 $\displaystyle\int \frac{dx}{x^2(ax+b)^2} = \frac{-a}{b^2(ax+b)} - \frac{1}{b^2x} + \frac{2a}{b^3}\ln\left(\frac{ax+b}{x}\right)$

72 $\displaystyle\int \frac{dx}{x^3(ax+b)^2} = -\frac{(ax+b)^2}{2b^4x^2} - \frac{a^3x}{b^4(ax+b)} + \frac{3a(ax+b)}{b^4x} - \frac{3a^2}{b^4}\ln\left(\frac{ax+b}{x}\right)$

73 $\displaystyle\int \frac{dx}{(ax+b)^3} = \frac{-1}{2(ax+b)^2}$

74 $\displaystyle\int \frac{x\,dx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$

75 $\displaystyle\int \frac{x^2\,dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3}\ln(ax+b)$

76 $\displaystyle\int \frac{x^3\,dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)^2} - \frac{3b}{a^4}\ln(ax+b)$

77 $\displaystyle\int \frac{dx}{x(ax+b)^3} = \frac{a^2x^2}{2b^3(ax+b)^2} - \frac{2ax}{b^3(ax+b)} - \frac{1}{b^3}\ln\left(\frac{ax+b}{x}\right)$

78 $\displaystyle\int \frac{dx}{x^2(ax+b)^3} = \frac{-a}{2b^2(ax+b)^2} - \frac{2a}{b^3(ax+b)} - \frac{1}{b^3x} + \frac{3a}{b^4}\ln\left(\frac{ax+b}{x}\right)$

79 $\displaystyle\int \frac{dx}{x^3(ax+b)^3} = \frac{a^4x^2}{2b^5(ax+b)^2} - \frac{4a^3x}{b^5(ax+b)} - \frac{(ax+b)^2}{2b^5x^2} - \frac{6a^2}{b^5}\ln\left(\frac{ax+b}{x}\right)$

80 $\displaystyle\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a};$ Si $n = -1$ véase 59.

81 $\displaystyle\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+2}}{(n+2)a^2} - \frac{b(ax+b)^{n+1}}{(n+1)a^2};$ $n \neq -1, -2$. Si $n = -1$ ó -2 véase 62 ó 67, respectivamente.

82 $\displaystyle\int x^2(ax+b)^n\,dx = \frac{(ax+b)^{n+3}}{(n+3)a^3} - \frac{2b(ax+b)^{n+2}}{(n+2)a^3} + \frac{b^2(ax+b)^{n+1}}{(n+1)a^3}.$ $\qquad n \neq -1, -2, -3.$

Si $n = -1, -2$ ó -3 véase 61, 68 ó 75, respectivamente.

83 $$\int x^m(ax+b)^n\,dx = \begin{cases} \dfrac{x^{m+1}(ax+b)^n}{m+n+1} + \dfrac{nb}{m+n+1}\displaystyle\int x^m(ax+b)^{n-1}\,dx \\[2ex] \dfrac{x^m(ax+b)^{n+1}}{(m+n+1)a} - \dfrac{mb}{(m+n+1)a}\displaystyle\int x^{m-1}(ax+b)^n\,dx \\[2ex] \dfrac{-x^{m+1}(ax+b)^{n+1}}{(n+1)b} + \dfrac{m+n+2}{(n+1)b}\displaystyle\int x^m(ax+b)^{n+1}\,dx \end{cases}$$

INTEGRALES CON $\sqrt{ax+b}$

84 $\displaystyle\int \frac{dx}{\sqrt{ax+b}} = \frac{2\sqrt{ax+b}}{a}$

85 $\displaystyle\int \frac{x\,dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b}$

86 $\displaystyle\int \frac{x^2\,dx}{\sqrt{ax+b}} = \frac{2(3a^2x^2-4abx+8b^2)}{15a^2}\sqrt{ax+b}$

87 $$\int \frac{dx}{x\sqrt{ax+b}} = \begin{cases} \dfrac{1}{b}\ln\left(\dfrac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right) \\[2ex] \dfrac{2}{\sqrt{-b}}\operatorname{tg}^{-1}\sqrt{\dfrac{ax+b}{-b}} \end{cases} \qquad b \neq 0$$

88 $\displaystyle\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int\frac{dx}{x\sqrt{ax+b}}$ $\qquad$ Véase 87 $\; b \neq 0$

89 $$\int \sqrt{ax+b}\,dx = \frac{2\sqrt{(ax+b)^3}}{3a}$$

90 $$\int x\sqrt{ax+b}\,dx = \frac{2(3ax-2b)}{15a^2}\sqrt{(ax+b)^3}$$

91 $$\int x^2\sqrt{ax+b}\,dx = \frac{2(15a^2x^2-12abx+8b^2)}{105a^3}\sqrt{(ax+b)^3}$$

92 $$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$$ Véase 87

93 $$\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}$$ Véase 87

94 $$\int \frac{x^m\,dx}{\sqrt{ax+b}} = \frac{2x^m\sqrt{ax+b}}{(2m+1)a} - \frac{2mb}{(2m+1)a}\int \frac{x^{m-1}\,dx}{\sqrt{ax+b}}$$

95 $$\int \frac{dx}{x^m\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{(m-1)b\,x^{m-1}} - \frac{(2m-3)a}{(2m-2)b}\int \frac{dx}{x^{m-1}\sqrt{ax+b}} \qquad m \neq 1$$

96 $$\int x^m\sqrt{ax+b}\,dx = \frac{2x^m}{(2m+3)a}\sqrt{(ax+b)^3} - \frac{2mb}{(2m+3)a}\int x^{m-1}\sqrt{ax+b}\,dx$$

97 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{ax+b}}{(m-1)x^{m-1}} + \frac{a}{2(m-1)}\int \frac{dx}{x^{m-1}\sqrt{ax+b}}$$

98 $$\int \frac{\sqrt{ax+b}}{x^m}\,dx = -\frac{\sqrt{(ax+b)^3}}{(m-1)b\,x^{m-1}} - \frac{(2m-5)a}{(2m-2)b}\int \frac{\sqrt{ax+b}}{x^{m-1}}\,dx$$

99 $$\int \sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+2}}}{a(m+2)}$$

100 $$\int x\sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+4}}}{a^2(m+4)} - \frac{2b\sqrt{(ax+b)^{m+2}}}{a^2(m+2)}$$

101 $$\int x^2\sqrt{(ax+b)^m}\,dx = \frac{2\sqrt{(ax+b)^{m+6}}}{a^3(m+6)} - \frac{4b\sqrt{(ax+b)^{m+4}}}{a^3(m+4)} + \frac{2b^2\sqrt{(ax+b)^{m+2}}}{a^3(m+2)}$$

102 $$\int \frac{\sqrt{(ax+b)^m}}{x}\,dx = 2\frac{\sqrt{(ax+b)^m}}{m} + b\int \frac{\sqrt{(ax+b)^{m-2}}}{x}\,dx$$

103 $$\int \frac{\sqrt{(ax+b)^m}}{x^2}\,dx = -\frac{\sqrt{(ax+b)^{m+2}}}{bx} + \frac{ma}{2b}\int \frac{\sqrt{(ax+b)^m}}{x}\,dx$$

104 $$\int \frac{dx}{x\sqrt{(ax+b)^m}} = \frac{2}{(m-2)b\sqrt{(ax+b)^{m-2}}} + \frac{1}{b}\int \frac{dx}{x^m\sqrt{(ax+b)^{m-2}}}$$

INTEGRALES CON $ax+b$ y $px+q$, donde $bp-aq \neq 0$

105 $$\int \frac{1}{(ax+b)(px+q)}\,dx = \frac{1}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)$$

106 $$\int \frac{x}{(ax+b)(px+q)}\,dx = \frac{1}{bp-aq}\left\{\frac{b}{a}\ln(ax+b) - \frac{q}{p}\ln(px+q)\right\}$$

107 $$\int \frac{1}{(ax+b)^2(px+q)}\,dx = \frac{1}{bp-aq}\left\{\frac{1}{ax+b} + \frac{p}{bp-aq}\ln\left(\frac{px+q}{ax+b}\right)\right\}$$

108 $$\int \frac{x}{(ax+b)^2(px+q)}\,dx = \frac{1}{bp-aq}\left\{\frac{q}{bp-aq}\ln\left(\frac{ax+b}{px+q}\right) - \frac{b}{a(ax+b)}\right\}$$

109 $$\int \frac{x^2\,dx}{(ax+b)^2(px+q)} = \frac{b^2}{(bp-aq)a^2(ax+b)} + \frac{1}{(bp-aq)^2}\left[\frac{q^2}{p}\ln(px+q) + \frac{b(bp-aq)}{a^2}\ln(ax+b)\right]$$

110 $$\int \frac{dx}{(ax+b)^m(px+q)^n} = \frac{-1}{(n-1)(bp-aq)}\left\{\frac{1}{(ax+b)^{m-1}(px+q)^{n-1}} + a(m+n-2)\int \frac{dx}{(ax+b)^m(px+q)^{n-1}}\right\}$$

111 $$\int \frac{ax+b}{px+q}\,dx = \frac{ax}{p} + \frac{bp-aq}{p^2}\ln(px+q)$$

112 $$\int \frac{(ax+b)^m}{(px+q)^n}\,dx = \begin{cases} \dfrac{-1}{(n-1)(bp-aq)}\left\{\dfrac{(ax+b)^{m+1}}{(px+q)^{n-1}} + a(n-m-2)\displaystyle\int \dfrac{(ax+b)^m}{(px+q)^{n-1}}\,dx\right\} \\ \dfrac{-1}{(n-m-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} + m(bp-aq)\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^n}\,dx\right\} \\ \dfrac{-1}{(n-1)p}\left\{\dfrac{(ax+b)^m}{(px+q)^{n-1}} - ma\displaystyle\int \dfrac{(ax+b)^{m-1}}{(px+q)^{n-1}}\,dx\right\} \end{cases}$$

$m, n \in \mathbb{N}$

INTEGRALES CON $\sqrt{ax+b}$ y $px+q$

113 $$\int \frac{px+q}{\sqrt{ax+b}}\,dx = \frac{2(apx+3aq-2bp)}{3a^2}\sqrt{ax+b}$$

114 $$\int \frac{dx}{(px+q)\sqrt{ax+b}} = \begin{cases} \dfrac{1}{\sqrt{bp-aq}\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2}{\sqrt{aq-bp}\sqrt{p}}\,tg^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

115 $$\int \frac{\sqrt{ax+b}}{px+q}\,dx = \begin{cases} \dfrac{2\sqrt{ax+b}}{p} + \dfrac{\sqrt{bp-aq}}{p\sqrt{p}}\ln\left(\dfrac{\sqrt{p(ax+b)}-\sqrt{bp-aq}}{\sqrt{p(ax+b)}+\sqrt{bp-aq}}\right) \\ \dfrac{2\sqrt{ax+b}}{p} - \dfrac{2\sqrt{aq-bp}}{p\sqrt{p}}\,tg^{-1}\sqrt{\dfrac{p(ax+b)}{aq-bp}} \end{cases}$$

116 $$\int (px+q)^n\sqrt{ax+b}\,dx = \frac{2(px+q)^{n+1}\sqrt{ax+b}}{(2n+3)p} + \frac{bp-aq}{(2n+3)p}\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx$$

117 $\displaystyle\int \frac{dx}{(px+q)^n\sqrt{ax+b}} = \frac{\sqrt{ax+b}}{(n-1)(aq-bp)(px+q)^{n-1}} + \frac{(2n-3)\,a}{2(n-1)(aq-bp)}\int\frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$

118 $\displaystyle\int \frac{(px+q)^n}{\sqrt{ax+b}}\,dx = \frac{2\,(px+q)^n\sqrt{ax+b}}{(2\,n+1)\,a} + \frac{2\,n\,(aq-bp)}{(2\,n+1)\,a}\int\frac{(px+q)^{n-1}}{\sqrt{ax+b}}\,dx$

119 $\displaystyle\int \frac{\sqrt{ax+b}}{(px+q)^n}\,dx = \frac{-\sqrt{ax+b}}{(n-1)\,p\,(px+q)^{n-1}} + \frac{a}{2\,(n-1)\,p}\int\frac{dx}{(px+q)^{n-1}\sqrt{ax+b}}$

INTEGRALES CON $\sqrt{ax+b}$ y $\sqrt{px+q}$

120 $\displaystyle\int \frac{dx}{\sqrt{(ax+b)(px+q)}} = \begin{cases} \dfrac{2}{\sqrt{ap}}\ln\left(\sqrt{a\,(px+q)}+\sqrt{p\,(ax+b)}\right) \\ \dfrac{2}{\sqrt{-ap}}\,tg^{-1}\sqrt{\dfrac{-p\,(ax+b)}{a\,(px+q)}} \end{cases}$

121 $\displaystyle\int \frac{x\,dx}{\sqrt{(ax+b)(px+q)}} = \frac{\sqrt{(ax+b)(px+q)}}{ap} - \frac{bp+aq}{2ap}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$

122 $\displaystyle\int \sqrt{(ax+b)(px+q)}\,dx = \frac{2apx+bp+aq}{4ap}\sqrt{(ax+b)(px+q)} - \frac{(bp-aq)^2}{8ap}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$

123 $\displaystyle\int \sqrt{\frac{px+q}{ax+b}}\,dx = \frac{\sqrt{(ax+b)(px+q)}}{a} + \frac{(aq-bp)}{2\,a}\int\frac{dx}{\sqrt{(ax+b)(px+q)}}$

124 $\displaystyle\int \frac{dx}{(px+q)\sqrt{(ax+b)(px+q)}} = \frac{2\sqrt{ax+b}}{(aq-bp)\sqrt{px+q}}$

INTEGRALES CON x^2+a^2

125 $\displaystyle\int \frac{dx}{x^2+a^2} = \frac{1}{a}\,tg^{-1}\frac{x}{a}$

126 $\displaystyle\int \frac{x\,dx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2)$

127 $\displaystyle\int \frac{x^2\,dx}{x^2+a^2} = x - a\,tg^{-1}\frac{x}{a}$

128 $\displaystyle\int \frac{x^3\,dx}{x^2+a^2} = \frac{x^2}{2} - \frac{a^2}{2}\ln(x^2+a^2)$

129 $\displaystyle\int \frac{dx}{x\,(x^2+a^2)} = \frac{1}{2\,a^2}\ln\left(\frac{x^2}{x^2+a^2}\right)$

130 $\displaystyle\int \frac{dx}{x^2\,(x^2+a^2)} = -\frac{1}{a^2\,x} - \frac{1}{a^3}\,tg^{-1}\frac{x}{a}$

131 $\displaystyle\int \frac{dx}{x^3\,(x^2+a^2)} = -\frac{1}{2\,a^2\,x^2} - \frac{1}{2\,a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$

132 $\displaystyle\int \frac{dx}{(x^2+a^2)^2} = \frac{x}{2\,a^2\,(x^2+a^2)} + \frac{1}{2\,a^3}\,tg^{-1}\frac{x}{a}$

133 $\displaystyle\int \frac{x\,dx}{(x^2+a^2)^2} = \frac{-1}{2\,(x^2+a^2)}$

134 $\displaystyle\int \frac{x^2\,dx}{(x^2+a^2)^2} = \frac{-x}{2\,(x^2+a^2)} + \frac{1}{2\,a}\,tg^{-1}\frac{x}{a}$

135 $\displaystyle\int \frac{x^3\,dx}{(x^2+a^2)^2} = \frac{a^2}{2\,(x^2+a^2)} + \frac{1}{2}\ln(x^2+a^2)$

136 $\displaystyle\int \frac{dx}{x\,(x^2+a^2)^2} = \frac{1}{2\,a^2\,(x^2+a^2)} + \frac{1}{2\,a^4}\ln\left(\frac{x^2}{x^2+a^2}\right)$

137 $\displaystyle\int \frac{dx}{x^2\,(x^2+a^2)^2} = -\frac{1}{a^4\,x} - \frac{x}{2\,a^4\,(x^2+a^2)} - \frac{3}{2\,a^5}\,tg^{-1}\frac{x}{a}$

138 $\displaystyle\int \frac{dx}{x^3\,(x^2+a^2)^2} = -\frac{1}{2\,a^4\,x^2} - \frac{1}{2\,a^4\,(x^2+a^2)} - \frac{1}{a^6}\ln\left(\frac{x^2}{x^2+a^2}\right)$

139 $\displaystyle\int \frac{dx}{(x^2+a^2)^n} = \frac{x}{2\,(n-1)\,a^2\,(x^2+a^2)^{n-1}} + \frac{2n-3}{(2n-2)\,a^2}\int\frac{dx}{(x^2+a^2)^{n-1}}$; si $n=1$ ver 125

140 $\displaystyle\int \frac{x\,dx}{(x^2+a^2)^n} = \frac{-1}{2\,(n-1)\,(x^2+a^2)^{n-1}}$; si $n=1$ ver 126

141 $\displaystyle\int \frac{dx}{x\,(x^2+a^2)^n} = \frac{1}{2\,(n-1)\,a^2\,(x^2+a^2)^{n-1}} + \frac{1}{a^2}\int\frac{dx}{x\,(x^2+a^2)^{n-1}}$; si $n=1$ ver 129

142 $\displaystyle\int \frac{x^m\,dx}{(x^2+a^2)^n} = \int\frac{x^{m-2}\,dx}{(x^2+a^2)^{n-1}} - a^2\int\frac{x^{m-2}\,dx}{(x^2+a^2)^n}$

143 $\displaystyle\int \frac{dx}{x^m\,(x^2+a^2)^n} = \frac{1}{a^2}\int\frac{dx}{x^m\,(x^2+a^2)^{n-1}} - \frac{1}{a^2}\int\frac{dx}{x^{m-2}\,(x^2+a^2)^n}$

INTEGRALES CON x^2-a^2, $x^2>a^2$

144 $\displaystyle\int \frac{dx}{x^2-a^2} = \frac{1}{2\,a}\ln\left(\frac{x-a}{x+a}\right)$ ó $-\dfrac{1}{a}\,cotgh^{-1}\dfrac{x}{a}$

145 $\displaystyle\int \frac{x\,dx}{x^2-a^2} = \frac{1}{2}\ln(x^2-a^2)$

146 $\displaystyle\int \frac{x^2\,dx}{x^2-a^2} = x + \frac{a}{2}\ln\left(\frac{x-a}{x+a}\right)$

147 $\displaystyle\int \frac{x^3\,dx}{x^2-a^2} = \frac{x^2}{2} + \frac{a^2}{2}\ln(x^2-a^2)$

148 $\displaystyle\int \frac{dx}{x\,(x^2-a^2)} = \frac{1}{2\,a^2}\ln\left(\frac{x^2-a^2}{x^2}\right)$

149 $\displaystyle\int \frac{dx}{x^2\,(x^2-a^2)} = \frac{1}{a^2\,x} + \frac{1}{2\,a^3}\ln\left(\frac{x-a}{x+a}\right)$

150 $\displaystyle\int \frac{dx}{x^3\,(x^2-a^2)} = \frac{1}{2\,a^2\,x^2} - \frac{1}{2\,a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

151 $\displaystyle\int \frac{dx}{(x^2-a^2)^2} = \frac{-x}{2\,a^2\,(x^2-a^2)} - \frac{1}{4\,a^3}\ln\left(\frac{x-a}{x+a}\right)$

152 $\int \frac{x\,dx}{(x^2-a^2)^2} = \frac{-1}{2\,(x^2-a^2)}$

153 $\int \frac{x^2\,dx}{(x^2-a^2)^2} = \frac{-x}{2\,(x^2-a^2)} + \frac{1}{4\,a}\ln\left(\frac{x-a}{x+a}\right)$

154 $\int \frac{x^3\,dx}{(x^2-a^2)^2} = \frac{-a^2}{2\,(x^2-a^2)} + \frac{1}{2}\ln(x^2-a^2)$

155 $\int \frac{dx}{x\,(x^2-a^2)^2} = \frac{-1}{2\,a^2\,(x^2-a^2)} + \frac{1}{2\,a^4}\ln\left(\frac{x^2}{x^2-a^2}\right)$

156 $\int \frac{dx}{x^2\,(x^2-a^2)^2} = -\frac{1}{a^4\,x} - \frac{x}{2\,a^4\,(x^2-a^2)} - \frac{3}{4\,a^5}\ln\left(\frac{x-a}{x+a}\right)$

157 $\int \frac{dx}{x^3\,(x^2-a^2)^2} = -\frac{1}{2\,a^4\,x^2} - \frac{1}{2\,a^4\,(x^2-a^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{x^2-a^2}\right)$

158 $\int \frac{dx}{(x^2-a^2)^n} = \frac{-x}{2\,(n-1)\,a^2\,(x^2-a^2)^{n-1}} - \frac{2n-3}{(2n-2)\,a^2}\int \frac{dx}{(x^2-a^2)^{n-1}}$

159 $\int \frac{x\,dx}{(x^2-a^2)^n} = \frac{-1}{2\,(n-1)\,(x^2-a^2)^{n-1}}$

160 $\int \frac{dx}{x\,(x^2-a^2)^n} = \frac{-1}{2\,(n-1)\,a^2\,(x^2-a^2)^{n-1}} - \frac{1}{a^2}\int \frac{dx}{x\,(x^2-a^2)^{n-1}}$

161 $\int \frac{x^m\,dx}{(x^2-a^2)^n} = \int \frac{x^{m-2}\,dx}{(x^2-a^2)^{n-1}} + a^2\int \frac{x^{m-2}\,dx}{(x^2-a^2)^n}$

162 $\int \frac{dx}{x^m\,(x^2-a^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}\,(x^2-a^2)^n} - \frac{1}{a^2}\int \frac{dx}{x^m\,(x^2-a^2)^{n-1}}$

INTEGRALES CON $a^2 - x^2,\ x^2 < a^2$

163 $\int \frac{dx}{a^2-x^2} = \frac{1}{2\,a}\ln\left(\frac{a+x}{a-x}\right)$ ó $\frac{1}{a}\,tgh^{-1}\frac{x}{a}$

164 $\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln(a^2-x^2)$

165 $\int \frac{x^2\,dx}{a^2-x^2} = -x + \frac{a}{2}\ln\left(\frac{a+x}{a-x}\right)$

166 $\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$

167 $\int \frac{dx}{x\,(a^2-x^2)} = \frac{1}{2\,a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$

168 $\int \frac{dx}{x^2\,(a^2-x^2)} = -\frac{1}{a^2\,x} + \frac{1}{2\,a^3}\ln\left(\frac{a+x}{a-x}\right)$

169 $\int \frac{dx}{x^3\,(a^2-x^2)} = -\frac{1}{2\,a^2\,x^2} + \frac{1}{2\,a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$

170 $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2\,a^2\,(a^2-x^2)} + \frac{1}{4\,a^3}\ln\left(\frac{a+x}{a-x}\right)$

171 $\int \frac{x\,dx}{(a^2-x^2)^2} = \frac{1}{2\,(a^2-x^2)}$

172 $\int \frac{x^2\,dx}{(a^2-x^2)^2} = \frac{x}{2\,(a^2-x^2)} - \frac{1}{4\,a}\ln\left(\frac{a+x}{a-x}\right)$

173 $\int \frac{x^3\,dx}{(a^2-x^2)^2} = \frac{a^2}{2\,(a^2-x^2)} + \frac{1}{2}\ln(a^2-x^2)$

174 $\int \frac{dx}{x\,(a^2-x^2)^2} = \frac{1}{2\,a^2\,(a^2-x^2)} + \frac{1}{2\,a^4}\ln\left(\frac{x^2}{a^2-x^2}\right)$

175 $\int \frac{dx}{x^2\,(a^2-x^2)^2} = -\frac{1}{a^4\,x} + \frac{x}{2\,a^4\,(a^2-x^2)} + \frac{3}{4\,a^5}\ln\left(\frac{a+x}{a-x}\right)$

176 $\int \frac{dx}{x^3\,(a^2-x^2)^2} = -\frac{1}{2\,a^4\,x^2} + \frac{1}{2\,a^4\,(a^2-x^2)} + \frac{1}{a^6}\ln\left(\frac{x^2}{a^2-x^2}\right)$

177 $\int \frac{dx}{(a^2-x^2)^n} = \frac{x}{2\,(n-1)\,a^2\,(a^2-x^2)^{n-1}} + \frac{2n-3}{(2n-2)\,a^2}\int \frac{dx}{(a^2-x^2)^{n-1}}$

178 $\int \frac{x\,dx}{(a^2-x^2)^n} = \frac{1}{2\,(n-1)\,(a^2-x^2)^{n-1}}$

179 $\int \frac{dx}{x\,(a^2-x^2)^n} = \frac{1}{2\,(n-1)\,a^2\,(a^2-x^2)^{n-1}} + \frac{1}{a^2}\int \frac{dx}{x\,(a^2-x^2)^{n-1}}$

180 $\int \frac{x^m\,dx}{(a^2-x^2)^n} = a^2\int \frac{x^{m-2}\,dx}{(a^2-x^2)^n} - \int \frac{x^{m-2}\,dx}{(a^2-x^2)^{n-1}}$

181 $\int \frac{dx}{x^m\,(a^2-x^2)^n} = \frac{1}{a^2}\int \frac{dx}{x^{m-2}\,(a^2-x^2)^n} + \frac{1}{a^2}\int \frac{dx}{x^m\,(a^2-x^2)^{n-1}}$

INTEGRALES CON $\sqrt{x^2 + a^2}$

182 $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right)$ ó $senh^{-1}\frac{x}{a}$

183 $\int \frac{x\,dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$

$|J| = \rho^2 \,sen\,\theta$

184 $\int \frac{x^2\,dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln\left(x + \sqrt{x^2+a^2}\right)$

185 $\int \frac{x^3\,dx}{\sqrt{x^2+a^2}} = \frac{\sqrt{(x^2+a^2)^3}}{3} - a^2\sqrt{x^2+a^2}$

186 $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$

187 $\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2\,x}$

188 $\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2\,a^2\,x^2} + \frac{1}{2\,a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$

189 $\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln\left(x + \sqrt{x^2+a^2}\right)$

190 $$\int x\sqrt{x^2+a^2}\,dx = \frac{\sqrt{(x^2+a^2)^3}}{3}$$

191 $$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

192 $$\int x^3\sqrt{x^2+a^2}\,dx = \frac{\sqrt{(x^2+a^2)^5}}{5} - \frac{a^2\sqrt{(x^2+a^2)^3}}{3}$$

193 $$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

194 $$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

195 $$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

196 $$\int \frac{dx}{\sqrt{(x^2+a^2)^3}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

197 $$\int \frac{x\,dx}{\sqrt{(x^2+a^2)^3}} = \frac{-1}{\sqrt{x^2+a^2}}$$

198 $$\int \frac{x^2\,dx}{\sqrt{(x^2+a^2)^3}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln\left(x+\sqrt{x^2+a^2}\right)$$

199 $$\int \frac{x^3\,dx}{\sqrt{(x^2+a^2)^3}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

200 $$\int \frac{dx}{x\sqrt{(x^2+a^2)^3}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

201 $$\int \frac{dx}{x^2\sqrt{(x^2+a^2)^3}} = -\frac{\sqrt{x^2+a^2}}{a^4x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

202 $$\int \frac{dx}{x^3\sqrt{(x^2+a^2)^3}} = \frac{-1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

203 $$\int \sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^3}}{4} + \frac{3a^2x\sqrt{x^2+a^2}}{8} + \frac{3a^4}{8}\ln\left(x+\sqrt{x^2+a^2}\right)$$

204 $$\int x\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^5}}{5}$$

205 $$\int x^2\sqrt{(x^2+a^2)^3}\,dx = \frac{x\sqrt{(x^2+a^2)^5}}{6} - \frac{a^2x\sqrt{(x^2+a^2)^3}}{24} - \frac{a^4x\sqrt{x^2+a^2}}{16} - \frac{a^6}{16}\ln\left(x+\sqrt{x^2+a^2}\right)$$

206 $$\int x^3\sqrt{(x^2+a^2)^3}\,dx = \frac{\sqrt{(x^2+a^2)^7}}{7} - \frac{a^2\sqrt{(x^2+a^2)^5}}{5}$$

207 $$\int \frac{\sqrt{(x^2+a^2)^3}}{x}\,dx = \frac{\sqrt{(x^2+a^2)^3}}{3} + a^2\sqrt{x^2+a^2} - a^3\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

208 $$\int \frac{\sqrt{(x^2+a^2)^3}}{x^2}\,dx = -\frac{\sqrt{(x^2+a^2)^3}}{x} + \frac{3x\sqrt{x^2+a^2}}{2} + \frac{3}{2}a^2\ln\left(x+\sqrt{x^2+a^2}\right)$$

209 $$\int \frac{\sqrt{(x^2+a^2)^3}}{x^3}\,dx = -\frac{\sqrt{(x^2+a^2)^3}}{2x} + \frac{3\sqrt{x^2+a^2}}{2} - \frac{3}{2}a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

INTEGRALES CON $\sqrt{x^2-a^2}$

210 $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right)$$

211 $$\int \frac{x\,dx}{\sqrt{x^2-a^2}} = \sqrt{x^2-a^2}$$

212 $$\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x\sqrt{x^2-a^2}}{2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)$$

213 $$\int \frac{x^3\,dx}{\sqrt{x^2-a^2}} = \frac{\sqrt{(x^2-a^2)^3}}{3} + a^2\sqrt{x^2-a^2}$$

214 $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$$

215 $$\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x}$$

216 $$\int \frac{dx}{x^3\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{2a^2x^2} + \frac{1}{2a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

217 $$\int \sqrt{x^2-a^2}\,dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right)$$

218 $$\int x\sqrt{x^2-a^2}\,dx = \frac{\sqrt{(x^2-a^2)^3}}{3}$$

219 $$\int x^2\sqrt{x^2-a^2}\,dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} + \frac{a^2x\sqrt{x^2-a^2}}{8} - \frac{a^4}{8}\ln\left(x+\sqrt{x^2-a^2}\right)$$

220 $$\int x^3\sqrt{x^2-a^2}\,dx = \frac{\sqrt{(x^2-a^2)^5}}{5} + \frac{a^2\sqrt{(x^2-a^2)^3}}{3}$$

221 $$\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$$

222 $$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left(x+\sqrt{x^2-a^2}\right)$$

223 $$\int \frac{\sqrt{x^2-a^2}}{x^3}\,dx = -\frac{\sqrt{x^2-a^2}}{2x^2} + \frac{1}{2a}\sec^{-1}\left|\frac{x}{a}\right|$$

224 $$\int \frac{dx}{\sqrt{(x^2-a^2)^3}} = -\frac{x}{a^2\sqrt{x^2-a^2}}$$

225 $$\int \frac{x\,dx}{\sqrt{(x^2-a^2)^3}} = \frac{-1}{\sqrt{x^2-a^2}}$$

226 $$\int \frac{x^2\,dx}{\sqrt{(x^2-a^2)^3}} = \frac{-x}{\sqrt{x^2-a^2}} + \ln\left(x+\sqrt{x^2-a^2}\right)$$

227 $$\int \frac{x^3\,dx}{\sqrt{(x^2-a^2)^3}} = \sqrt{x^2-a^2} - \frac{a^2}{\sqrt{x^2-a^2}}$$

228 $$\int \frac{dx}{x\sqrt{(x^2-a^2)^3}} = \frac{-1}{a^2\sqrt{x^2-a^2}} - \frac{1}{a^3}\sec^{-1}\left|\frac{x}{a}\right|$$

229 $$\int \frac{dx}{x^2\sqrt{(x^2-a^2)^3}} = -\frac{\sqrt{x^2-a^2}}{a^4 x} - \frac{x}{a^4\sqrt{x^2-a^2}}$$

230 $$\int \frac{dx}{x^3\sqrt{(x^2-a^2)^3}} = \frac{1}{2a^2x^2\sqrt{x^2-a^2}} - \frac{3}{2a^4\sqrt{x^2-a^2}} - \frac{3}{2a^5}\sec^{-1}\left|\frac{x}{a}\right|$$

231 $$\int \sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^3}}{4} - \frac{3a^2x\sqrt{x^2-a^2}}{8} + \frac{3a^4}{8}\ln\left(x+\sqrt{x^2-a^2}\right)$$

232 $$\int x\sqrt{(x^2-a^2)^3}\,dx = \frac{\sqrt{(x^2-a^2)^5}}{5}$$

233 $$\int x^2\sqrt{(x^2-a^2)^3}\,dx = \frac{x\sqrt{(x^2-a^2)^5}}{6} + \frac{a^2x\sqrt{(x^2-a^2)^3}}{24} - \frac{a^4x\sqrt{x^2-a^2}}{16} + \frac{a^6}{16}\ln\left(x+\sqrt{x^2-a^2}\right)$$

234 $$\int x^3\sqrt{(x^2-a^2)^3}\,dx = \frac{\sqrt{(x^2-a^2)^7}}{7} + \frac{a^2\sqrt{(x^2-a^2)^5}}{5}$$

235 $$\int \frac{\sqrt{(x^2-a^2)^3}}{x}\,dx = \frac{\sqrt{(x^2-a^2)^3}}{3} - a^2\sqrt{x^2-a^2} + a^3\sec^{-1}\left|\frac{x}{a}\right|$$

236 $$\int \frac{\sqrt{(x^2-a^2)^3}}{x^2}\,dx = -\frac{\sqrt{(x^2-a^2)^3}}{x} + \frac{3x\sqrt{x^2-a^2}}{2} - \frac{3}{2}a^2\ln\left(x+\sqrt{x^2-a^2}\right)$$

237 $$\int \frac{\sqrt{(x^2-a^2)^3}}{x^3}\,dx = -\frac{\sqrt{(x^2-a^2)^3}}{2x^2} + \frac{3\sqrt{x^2-a^2}}{2} - \frac{3}{2}a\sec^{-1}\left|\frac{x}{a}\right|$$

INTEGRALES CON $\sqrt{a^2-x^2}$

238 $$\int \frac{dx}{\sqrt{a^2-x^2}} = \operatorname{sen}^{-1}\frac{x}{a}$$

239 $$\int \frac{x\,dx}{\sqrt{a^2-x^2}} = -\sqrt{a^2-x^2}$$

240 $$\int \frac{x^2\,dx}{\sqrt{a^2-x^2}} = -\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$$

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241 $$\int \frac{x^3\,dx}{\sqrt{a^2-x^2}} = \frac{\sqrt{(a^2-x^2)^3}}{3} - a^2\sqrt{a^2-x^2}$$

242 $$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

243 $$\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x}$$

244 $$\int \frac{dx}{x^3\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{2a^2x^2} - \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

245 $$\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{x}{a}$$

246 $$\int x\sqrt{a^2-x^2}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{3}$$

247 $$\int x^2\sqrt{a^2-x^2}\,dx = -\frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{a^2x\sqrt{a^2-x^2}}{8} - \frac{a^4}{8}\operatorname{sen}^{-1}\frac{x}{a}$$

248 $$\int x^3\sqrt{a^2-x^2}\,dx = \frac{\sqrt{(a^2-x^2)^5}}{5} - \frac{a^2\sqrt{(a^2-x^2)^3}}{3}$$

249 $$\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

250 $$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{a^2-x^2}}{x} - \operatorname{sen}^{-1}\frac{x}{a}$$

251 $$\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{a^2-x^2}}{2x^2} + \frac{1}{2a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

252 $$\int \frac{dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{a^2\sqrt{a^2-x^2}}$$

253 $$\int \frac{x\,dx}{\sqrt{(a^2-x^2)^3}} = \frac{1}{\sqrt{a^2-x^2}}$$

254 $$\int \frac{x^2\,dx}{\sqrt{(a^2-x^2)^3}} = \frac{x}{\sqrt{a^2-x^2}} - \operatorname{sen}^{-1}\frac{x}{a}$$

255 $$\int \frac{x^3\,dx}{\sqrt{(a^2-x^2)^3}} = \sqrt{a^2-x^2} + \frac{a^2}{\sqrt{a^2-x^2}}$$

256 $$\int \frac{dx}{x\sqrt{(a^2-x^2)^3}} = \frac{1}{a^2\sqrt{a^2-x^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

257 $$\int \frac{dx}{x^2\sqrt{(a^2-x^2)^3}} = -\frac{\sqrt{a^2-x^2}}{a^4x} + \frac{x}{a^4\sqrt{a^2-x^2}}$$

258 $$\int \frac{dx}{x^3\sqrt{(a^2-x^2)^3}} = \frac{-1}{2a^2x^2\sqrt{a^2-x^2}} + \frac{3}{2a^4\sqrt{a^2-x^2}} - \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

259 $$\int \sqrt{(a^2-x^2)^3}\,dx = \frac{x\sqrt{(a^2-x^2)^3}}{4} + \frac{3a^2x\sqrt{a^2-x^2}}{8} + \frac{3a^4}{8}\,\mathrm{sen}^{-1}\frac{x}{a}$$

260 $$\int x\sqrt{(a^2-x^2)^3}\,dx = -\frac{\sqrt{(a^2-x^2)^5}}{5}$$

261 $$\int x^2\sqrt{(a^2-x^2)^3}\,dx = -\frac{x\sqrt{(a^2-x^2)^5}}{6} + \frac{a^2x\sqrt{(a^2-x^2)^3}}{24} + \frac{a^4x\sqrt{a^2-x^2}}{16} + \frac{a^6}{16}\,\mathrm{sen}^{-1}\frac{x}{a}$$

262 $$\int x^3\sqrt{(a^2-x^2)^3}\,dx = \frac{\sqrt{(a^2-x^2)^7}}{7} - \frac{a^2\sqrt{(a^2-x^2)^5}}{5}$$

263 $$\int \frac{\sqrt{(a^2-x^2)^3}}{x}\,dx = \frac{\sqrt{(a^2-x^2)^3}}{3} + a^2\sqrt{a^2-x^2} - a^3\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

264 $$\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{x} - \frac{3x\sqrt{a^2-x^2}}{2} - \frac{3}{2}a^2\,\mathrm{sen}^{-1}\frac{x}{a}$$

265 $$\int \frac{\sqrt{a^2-x^2}}{x^3}\,dx = -\frac{\sqrt{(a^2-x^2)^3}}{2x^2} - \frac{3\sqrt{a^2-x^2}}{2} + \frac{3}{2}a\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$$

INTEGRALES CON $ax^2 + bx + c$

Si $b^2 = 4ac$, se puede escribir $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2$ y se emplean los resultados de las páginas 11 y 12.

266 $$\int \frac{dx}{ax^2+bx+c} = \begin{cases} \dfrac{2}{\sqrt{4ac-b^2}}\,\mathrm{tg}^{-1}\dfrac{2ax+b}{\sqrt{4ac-b^2}} \\[2ex] \dfrac{1}{\sqrt{b^2-4ac}}\ln\left(\dfrac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right) \end{cases}$$

267 $$\int \frac{x\,dx}{ax^2+bx+c} = \frac{1}{2a}\ln(ax^2+bx+c) - \frac{b}{2a}\int\frac{dx}{ax^2+bx+c}$$

268 $$\int \frac{x^2\,dx}{ax^2+bx+c} = \frac{x}{a} - \frac{b}{2a^2}\ln(ax^2+bx+c) + \frac{b^2-2ac}{2a^2}\int\frac{dx}{ax^2+bx+c}$$

269 $$\int \frac{x^m\,dx}{ax^2+bx+c} = \frac{x^{m-1}}{(m-1)a} - \frac{c}{a}\int\frac{x^{m-2}\,dx}{ax^2+bx+c} - \frac{b}{a}\int\frac{x^{m-1}\,dx}{ax^2+bx+c}$$

270 $$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left(\frac{x^2}{ax^2+bx+c}\right) - \frac{b}{2c}\int\frac{dx}{ax^2+bx+c}$$

271 $$\int \frac{dx}{x^2(ax^2+bx+c)} = \frac{b}{2c^2}\ln\left(\frac{ax^2+bx+c}{x^2}\right) - \frac{1}{xc} + \frac{b^2-2ac}{2c^2}\int\frac{dx}{ax^2+bx+c}$$

272 $$\int \frac{dx}{x^n(ax^2+bx+c)} = -\frac{1}{(n-1)c\,x^{n-1}} - \frac{b}{c}\int\frac{dx}{x^{n-1}(ax^2+bx+c)} - \frac{a}{c}\int\frac{dx}{x^{n-2}(ax^2+bx+c)}$$

273 $$\int \frac{dx}{(ax^2+bx+c)^2} = \frac{2ax+b}{(4ac-b^2)(ax^2+bx+c)} + \frac{2a}{4ac-b^2}\int\frac{dx}{ax^2+bx+c}$$

274 $$\int \frac{x\,dx}{(ax^2+bx+c)^2} = -\frac{bx+2c}{(4ac-b^2)(ax^2+bx+c)} - \frac{b}{4ac-b^2}\int\frac{dx}{ax^2+bx+c}$$

275 $$\int \frac{x^2\,dx}{(ax^2+bx+c)^2} = \frac{(b^2-2ac)x+bc}{a(4ac-b^2)(ax^2+bx+c)} + \frac{2c}{4ac-b^2}\int\frac{dx}{ax^2+bx+c}$$

276 $$\int \frac{x^m\,dx}{(ax^2+bx+c)^n} = \frac{-x^{m-1}}{(2n-m-1)a(ax^2+bx+c)^{n-1}} + \frac{1}{(2n-m-1)a}\left\{c\int\frac{(m-1)x^{m-2}dx}{(ax^2+bx+c)^n} - b\int\frac{(n-m)x^{m-1}dx}{(ax^2+bx+c)^n}\right\}$$

277 $$\int \frac{x^{2n-1}\,dx}{(ax^2+bx+c)^n} = \frac{1}{a}\int\frac{x^{2n-3}\,dx}{(ax^2+bx+c)^{n-1}} - \frac{c}{a}\int\frac{x^{2n-3}\,dx}{(ax^2+bx+c)^n} - \frac{b}{a}\int\frac{x^{2n-2}\,dx}{(ax^2+bx+c)^n}$$

278 $$\int \frac{dx}{x(ax^2+bx+c)^2} = \frac{1}{2c(ax^2+bx+c)} - \frac{b}{2c}\int\frac{dx}{(ax^2+bx+c)^2} + \frac{1}{c}\int\frac{dx}{x(ax^2+bx+c)}$$

279 $$\int \frac{dx}{x^2(ax^2+bx+c)^2} = \frac{-1}{cx(ax^2+bx+c)} - \frac{3a}{c}\int\frac{dx}{(ax^2+bx+c)^2} - \frac{2b}{c}\int\frac{dx}{x(ax^2+bx+c)^2}$$

280 $$\int \frac{dx}{x^m(ax^2+bx+c)^n} = \frac{-1}{(m-1)c\,x^{m-1}(ax^2+bx+c)^{n-1}}$$
$$- \frac{(m+2n-3)a}{(m-1)c}\int\frac{dx}{x^{m-2}(ax^2+bx+c)^n}$$
$$- \frac{(m+n-2)b}{(m-1)c}\int\frac{dx}{x^{m-1}(ax^2+bx+c)^n}$$

INTEGRALES CON $\sqrt{ax^2 + bx + c}$

Si en las fórmulas siguientes $b^2 = 4ac$, se puede escribir $\sqrt{ax^2+bx+c} = \sqrt{a}\left(x + \frac{b}{2a}\right)$ y se emplean los resultados de las páginas 11 y 12.

281 $$\int \frac{dx}{\sqrt{(ax^2+bx+c)}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left(2\sqrt{a}\sqrt{(ax^2+bx+c)} + 2ax + b\right) \\[2ex] -\dfrac{1}{\sqrt{-a}}\,\mathrm{sen}^{-1}\left(\dfrac{2ax+b}{\sqrt{b^2-4ac}}\right) \quad \text{ó} \quad \dfrac{1}{\sqrt{a}}\,\mathrm{senh}^{-1}\left(\dfrac{2ax+b}{\sqrt{4ac-b^2}}\right) \end{cases}$$

282 $$\int \frac{x\,dx}{\sqrt{ax^2+bx+c}} = \frac{\sqrt{ax^2+bx+c}}{a} - \frac{b}{2a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

283 $$\int \frac{x^2\,dx}{\sqrt{ax^2+bx+c}} = \frac{2ax-3b}{4a^2}\sqrt{ax^2+bx+c} + \frac{3b^2-4ac}{8a^2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

284 $$\int \frac{dx}{x\sqrt{ax^2+bx+c}} = \begin{cases} -\dfrac{1}{\sqrt{c}}\ln\left(\dfrac{2\sqrt{c}\sqrt{ax^2+bx+c}+bx+2c}{x}\right) \\[2ex] \dfrac{1}{\sqrt{-c}}\,\mathrm{sen}^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{b^2-4ac}}\right) \quad \text{ó} \quad -\dfrac{1}{\sqrt{c}}\,\mathrm{senh}^{-1}\left(\dfrac{bx+2c}{|x|\sqrt{4ac-b^2}}\right) \end{cases}$$

285 $$\int \frac{dx}{x^2\sqrt{ax^2+bx+c}} = -\frac{\sqrt{ax^2+bx+c}}{cx} - \frac{b}{2c}\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$

286 $$\int \sqrt{ax^2+bx+c}\,dx = \frac{2ax+b}{4a}\sqrt{(ax^2+bx+c)} + \frac{4ac-b^2}{8a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

287 $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{\sqrt{(ax^2+bx+c)^3}}{3a} - \frac{b\,(2ax+b)}{8a^2}\sqrt{ax^2+bx+c} - \frac{b\,(4ac-b^2)}{16a^2}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

288 $$\int x^2\sqrt{ax^2+bx+c}\,dx = \frac{8ax-5b}{24\,a^2}\sqrt{(ax^2+bx+c)^3} + \frac{5b^2-4ac}{16\,a^2}\int\sqrt{ax^2+bx+c}\,dx$$

289 $$\int\frac{\sqrt{ax^2+bx+c}}{x}\,dx = \sqrt{ax^2+bx+c} + \frac{b}{2}\int\frac{dx}{\sqrt{ax^2+bx+c}} + c\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$

290 $$\int\frac{\sqrt{ax^2+bx+c}}{x^2}\,dx = -\frac{\sqrt{ax^2+bx+c}}{x} + a\int\frac{dx}{\sqrt{ax^2+bx+c}} + \frac{b}{2}\int\frac{dx}{x\sqrt{ax^2+bx+c}}$$

291 $$\int\frac{dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{2\,(2ax+b)}{(4ac-b^2)\sqrt{ax^2+bx+c}}$$

292 $$\int\frac{x\,dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{2\,(bx+2c)}{(b^2-4ac)\sqrt{ax^2+bx+c}}$$

293 $$\int\frac{x^2\,dx}{\sqrt{(ax^2+bx+c)^3}} = \frac{(2b^2-4ac)\,x+2bc}{a\,(4ac-b^2)\sqrt{ax^2+bx+c}} + \frac{1}{a}\int\frac{dx}{\sqrt{ax^2+bx+c}}$$

294 $$\int\frac{dx}{x\sqrt{(ax^2+bx+c)^3}} = \frac{1}{c\sqrt{ax^2+bx+c}} + \frac{1}{c}\int\frac{dx}{x\sqrt{ax^2+bx+c}} - \frac{b}{2c}\int\frac{dx}{\sqrt{(ax^2+bx+c)^3}}$$

295 $$\int\frac{dx}{x^2\sqrt{(ax^2+bx+c)^3}} = \frac{-(ax^2+bx+c)}{c^2x\sqrt{ax^2+bx+c}} - \frac{3b}{2c^2}\int\frac{dx}{x\sqrt{ax^2+bx+c}} + \frac{b^2-4ac}{2c^2}\int\frac{dx}{\sqrt{(ax^2+bx+c)^3}}$$

296 $$\int\sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{(2ax+b)\sqrt{(ax^2+bx+c)^{n+1}}}{4\,a\,(n+1)} + \frac{(2n+1)(4ac-b^2)}{8\,a\,(n+1)}\int\sqrt{(ax^2+bx+c)^{n-1}}\,dx$$

297 $$\int x\sqrt{(ax^2+bx+c)^{n+1}}\,dx = \frac{\sqrt{(ax^2+bx+c)^{n+3}}}{a\,(2n+3)} - \frac{b}{2a}\int\sqrt{(ax^2+bx+c)^{n+1}}\,dx$$

298 $$\int\frac{dx}{\sqrt{(ax^2+bx+c)^{n+1}}} = \frac{1}{(2n-1)(4ac-b^2)}\left(\frac{2\,(2ax+b)}{\sqrt{(ax^2+bx+c)^{n-1}}} + 8\,a\,(n-1)\int\frac{dx}{\sqrt{(ax^2+bx+c)^{n-1}}}\right)$$

299 $$\int\frac{dx}{\sqrt{(ax^2+bx+c)^{n+1}}} = \frac{1}{(2n-1)\,c\sqrt{(ax^2+bx+c)^{n-1}}} + \frac{1}{c}\int\frac{dx}{x\sqrt{(ax^2+bx+c)^{n-1}}} - \frac{b}{2c}\int\frac{dx}{\sqrt{(ax^2+bx+c)^{n+1}}}$$

INTEGRALES CON $x^3 + a^3$

Para integrales con $x^3 - a^3$, se reemplaza a por -a

300 $$\int\frac{dx}{x^3+a^3} = \frac{1}{6\,a^2}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{1}{a^2\sqrt{3}}\,tg^{-1}\frac{2x-a}{a\sqrt{3}}$$

301 $$\int\frac{x\,dx}{x^3+a^3} = \frac{1}{6\,a}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{a\sqrt{3}}\,tg^{-1}\frac{2x-a}{a\sqrt{3}}$$

302 $$\int\frac{x^2\,dx}{x^3+a^3} = \frac{1}{3}\ln(x^3+a^3)$$

303 $$\int\frac{dx}{x\,(x^3+a^3)} = \frac{1}{3\,a^3}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

304 $$\int\frac{dx}{x^2\,(x^3+a^3)} = -\frac{1}{a^3\,x} - \frac{1}{6\,a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} - \frac{1}{a^4\sqrt{3}}\,tg^{-1}\frac{2x-a}{a\sqrt{3}}$$

305 $$\int\frac{dx}{(x^3+a^3)^2} = \frac{x}{3\,a^3\,(x^3+a^3)} + \frac{1}{9\,a^5}\ln\frac{(x+a)^2}{x^2-ax+a^2} + \frac{2}{3\,a^5\sqrt{3}}\,tg^{-1}\frac{2x-a}{a\sqrt{3}}$$

306 $$\int\frac{x\,dx}{(x^3+a^3)^2} = \frac{x^2}{3\,a^3\,(x^3+a^3)} + \frac{1}{18\,a^4}\ln\frac{x^2-ax+a^2}{(x+a)^2} + \frac{1}{3\,a^4\sqrt{3}}\,tg^{-1}\frac{2x-a}{a\sqrt{3}}$$

307 $$\int\frac{x^2\,dx}{(x^3+a^3)^2} = -\frac{1}{3\,(x^3+a^3)}$$

308 $$\int\frac{dx}{x\,(x^3+a^3)^2} = \frac{1}{3\,a^3\,(x^3+a^3)} + \frac{1}{3\,a^6}\ln\left(\frac{x^3}{x^3+a^3}\right)$$

309 $$\int\frac{dx}{x^2\,(x^3+a^3)^2} = -\frac{1}{a^6\,x} - \frac{x^2}{3\,a^6\,(x^3+a^3)} - \frac{4}{3\,a^6}\int\frac{x\,dx}{x^3+a^3}$$ *Véase 301*

310 $$\int\frac{x^m\,dx}{x^3+a^3} = \frac{x^{m-2}}{(m-2)} - a^3\int\frac{x^{m-3}\,dx}{x^3+a^3}$$

311 $$\int\frac{dx}{x^n\,(x^3+a^3)} = \frac{-1}{a^3\,(n-1)\,x^{n-1}} - \frac{1}{a^3}\int\frac{dx}{x^{n-3}\,(x^3+a^3)}$$

INTEGRALES CON $x^4 \pm a^4$

312 $$\int\frac{dx}{x^4+a^4} = \frac{1}{4\,a^3\sqrt{2}}\ln\left(\frac{x^2+a\,x\sqrt{2}+a^2}{x^2-a\,x\sqrt{2}+a^2}\right) - \frac{-1}{2\,a^3\sqrt{2}}\,tg^{-1}\frac{a\,x\sqrt{2}}{x^2-a^2}$$

313 $$\int\frac{x\,dx}{x^4+a^4} = \frac{1}{2\,a^2}\,tg^{-1}\frac{x^2}{a^2}$$

314 $$\int\frac{x^2\,dx}{x^4+a^4} = \frac{1}{4\,a\sqrt{2}}\ln\left(\frac{x^2-a\,x\sqrt{2}+a^2}{x^2+a\,x\sqrt{2}+a^2}\right) - \frac{1}{2\,a\sqrt{2}}\,tg^{-1}\frac{a\,x\sqrt{2}}{x^2-a^2}$$

315 $$\int\frac{x^3\,dx}{x^4+a^4} = \frac{1}{4}\ln(x^4+a^4)$$

316 $$\int\frac{dx}{x\,(x^4+a^4)} = \frac{1}{4\,a^4}\ln\left(\frac{x^4}{x^4+a^4}\right)$$

317 $$\int\frac{dx}{x^2\,(x^4+a^4)} = -\frac{1}{a^4\,x} - \frac{1}{4\,a^5\sqrt{2}}\ln\left(\frac{x^2-a\,x\sqrt{2}+a^2}{x^2+a\,x\sqrt{2}+a^2}\right) + \frac{1}{2\,a^5\sqrt{2}}\,tg^{-1}\frac{a\,x\sqrt{2}}{x^2-a^2}$$

318 $$\int\frac{dx}{x^3\,(x^4+a^4)} = -\frac{1}{2\,a^4\,x^2} - \frac{1}{2\,a^6}\,tg^{-1}\frac{x^2}{a^2}$$

319 $$\int\frac{dx}{x^4-a^4} = \frac{1}{4\,a^3}\ln\left(\frac{x-a}{x+a}\right) - \frac{1}{2\,a^3}\,tg^{-1}\frac{x}{a}$$

320 $$\int\frac{x\,dx}{x^4-a^4} = \frac{1}{4\,a^2}\ln\left(\frac{x^2-a^2}{x^2+a^2}\right)$$

321 $$\int\frac{x^2\,dx}{x^4-a^4} = \frac{1}{4\,a}\ln\left(\frac{x-a}{x+a}\right) + \frac{1}{2\,a}\,tg^{-1}\frac{x}{a}$$

322 $$\int \frac{x^3\,dx}{x^4 - a^4} = \frac{1}{4} \ln (x^4 - a^4)$$

323 $$\int \frac{dx}{x\,(x^4 - a^4)} = \frac{1}{4\,a^4} \ln \left(\frac{x^4 - a^4}{x^4} \right)$$

324 $$\int \frac{dx}{x^2\,(x^4 - a^4)} = \frac{1}{a^4\,x} + \frac{1}{4\,a^5} \ln \left(\frac{x - a}{x + a} \right) + \frac{1}{2\,a^5}\, tg^{-1} \frac{x}{a}$$

325 $$\int \frac{dx}{x^3\,(x^4 - a^4)} = \frac{1}{2\,a^4\,x^2} + \frac{1}{4\,a^6} \ln \left(\frac{x^2 - a^2}{x^2 + a^2} \right)$$

INTEGRALES CON $x^n \pm a^n$

326 $$\int \frac{dx}{x\,(x^n + a^n)} = \frac{1}{n\,a^n} \ln \frac{x^n}{x^n + a^n}$$

327 $$\int \frac{x^{n-1}\,dx}{x^n + a^n} = \frac{1}{n} \ln (x^n + a^n)$$

328 $$\int \frac{x^m\,dx}{(x^r + a^r)^n} = \int \frac{x^{m-r}\,dx}{(x^r + a^r)^{n-1}} - a^r \int \frac{x^{m-r}\,dx}{(x^r + a^r)^n}$$

329 $$\int \frac{dx}{x^m\,(x^r + a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^m\,(x^r + a^r)^{n-1}} - \frac{1}{a^r} \int \frac{dx}{x^{m-r}\,(x^r + a^r)^n}$$

330 $$\int \frac{dx}{x\,\sqrt{x^n + a^n}} = \frac{1}{n\,\sqrt{a^n}} \ln \left(\frac{\sqrt{x^n + a^n} - \sqrt{a^n}}{\sqrt{x^n + a^n} + \sqrt{a^n}} \right)$$

331 $$\int \frac{dx}{x\,(x^n - a^n)} = \frac{1}{n\,a^n} \ln \left(\frac{x^n - a^n}{x^n} \right)$$

332 $$\int \frac{x^{n-1}\,dx}{x^n - a^n} = \frac{1}{n} \ln (x^n - a^n)$$

333 $$\int \frac{x^m\,dx}{(x^r - a^r)^n} = a^r \int \frac{x^{m-r}\,dx}{(x^r - a^r)^n} + \int \frac{x^{m-r}\,dx}{(x^r - a^r)^{n-1}}$$

334 $$\int \frac{dx}{x^m\,(x^r - a^r)^n} = \frac{1}{a^r} \int \frac{dx}{x^{m-r}\,(x^r - a^r)^n} - \frac{1}{a^r} \int \frac{dx}{x^m\,(x^r - a^r)^{n-1}}$$

335 $$\int \frac{dx}{x\,\sqrt{x^n - a^n}} = \frac{2}{n\,\sqrt{a^n}} \cos^{-1} \sqrt{\frac{a^n}{x^n}}$$

336 $$\int \frac{x^{p-1}\,dx}{(x^{2m} + a^{2m})} = \frac{1}{m\,a^{2m-p}} \sum_{k=1}^{m} sen \left(\frac{(2k-1)\,p\,\pi}{2\,m} \right) \cdot tg^{-1} \left(\frac{x + a \cos \left(\frac{(2k-1)\,\pi}{2\,m} \right)}{a\, sen \left(\frac{(2k-1)\,\pi}{2\,m} \right)} \right)$$
$$- \frac{1}{2\,m\,a^{2m-p}} \sum_{k=1}^{m} \cos \left(\frac{(2k-1)\,p\,\pi}{2\,m} \right) \cdot \ln \left\{ x^2 + 2\,a\,x \cos \left(\frac{(2k-1)\,\pi}{2\,m} \right) + a^2 \right\}$$

337 $$\int \frac{x^{p-1}\,dx}{(x^{2m} - a^{2m})} = \frac{1}{2\,m\,a^{2m-p}} \sum_{k=1}^{m-1} \cos \frac{k\,p\,\pi}{m} \cdot \ln \left\{ x^2 - 2\,a\,x \cos \frac{k\,\pi}{m} + a^2 \right\}$$
$$- \frac{1}{m\,a^{2m-p}} \sum_{k=1}^{m-1} sen \frac{k\,p\,\pi}{m} \cdot tg^{-1} \left(\frac{x - a \cos \frac{k\,\pi}{m}}{a\, sen \frac{k\,\pi}{m}} \right)$$
$$+ \frac{1}{2\,m\,a^{2m-p}} \{ \ln (x - a) + (-1)^p \ln (x + a) \}$$

tanto en 336 como en 337 es $0 < p \leq 2\,m$.

338 $$\int \frac{x^{p-1}\,dx}{x^{2m+1} + a^{2m+1}} = \frac{2\,(-1)^{p-1}}{(2m + 1)\,a^{2m-p+1}} \sum_{k=1}^{m} sen \frac{2\,k\,p\,\pi}{2m + 1} \cdot tg^{-1} \left(\frac{x + a \cos \frac{2\,k\,\pi}{2m + 1}}{a\, sen \frac{2\,k\,\pi}{2m + 1}} \right)$$
$$- \frac{(-1)^{p-1}}{(2m + 1)\,a^{2m-p+1}} \sum_{k=1}^{m} \cos \frac{2\,k\,p\,\pi}{2m + 1} \cdot \ln \left\{ x^2 - 2\,a\,x \cos \frac{2\,k\,\pi}{2m + 1} + a^2 \right\}$$
$$+ \frac{(-1)^{p-1} \ln (x + a)}{(2m + 1)\,a^{2m-p+1}}$$

339 $$\int \frac{x^{p-1}\,dx}{x^{2m+1} - a^{2m+1}} = \frac{-2}{(2m + 1)\,a^{2m-p+1}} \sum_{k=1}^{m} sen \frac{2\,k\,p\,\pi}{2m + 1} \cdot tg^{-1} \left(\frac{x - a \cos \frac{2\,k\,\pi}{2m + 1}}{a\, sen \frac{2\,k\,\pi}{2m + 1}} \right)$$
$$+ \frac{1}{(2m + 1)\,a^{2m-p+1}} \sum_{k=1}^{m} \cos \frac{2\,k\,p\,\pi}{2m + 1} \cdot \ln \left\{ x^2 - 2\,a\,x \cos \frac{2\,k\,\pi}{2m + 1} + a^2 \right\}$$
$$+ \frac{\ln (x - a)}{(2m + 1)\,a^{2m-p+1}}$$

tanto en 338 como en 339 es $0 < p \leq 2\,m + 1$.

INTEGRALES CON sen ax

340 $$\int sen\, ax\, dx = - \frac{\cos ax}{a}$$

341 $$\int x\, sen\, ax\, dx = \frac{sen\, ax}{a^2} - \frac{x \cos ax}{a}$$

342 $$\int x^2\, sen\, ax\, dx = \frac{2x}{a^2}\, sen\, ax + \left(\frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

343 $$\int x^3\, sen\, ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4} \right) sen\, ax + \left(\frac{6x}{a^3} - \frac{x^3}{a} \right) \cos ax$$

344 $$\int \frac{sen\, ax}{x}\, dx = ax - \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,(ax)^{2n-1}}{(2n - 1)\,(2n - 1)!}$$

345 $$\int \frac{sen\, ax}{x^2}\, dx = - \frac{sen\, ax}{x} + a \int \frac{\cos ax}{x}\, dx$$ Véase 374

346 $$\int \frac{dx}{sen\, ax} = \frac{1}{a} \ln (cosec\, ax - cotg\, ax) = \frac{1}{a} \ln tg \frac{ax}{2}$$

347 $$\int \frac{x\,dx}{sen\, ax} = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7\,(ax)^5}{1800} + \cdots + \frac{2\,(2^{2n-1} - 1)\,B_n\,(ax)^{2n+1}}{(2n + 1)!} + \cdots \right\}$$ B_n es

nº de Bernoulli

348 $\int \operatorname{sen}^2 ax\, dx = \frac{x}{2} - \frac{\operatorname{sen} 2ax}{4a}$

349 $\int x \operatorname{sen}^2 ax\, dx = \frac{x^2}{4} - \frac{x \operatorname{sen} 2ax}{4a} - \frac{\cos 2ax}{8a^2}$

350 $\int \operatorname{sen}^3 ax\, dx = -\frac{\cos ax}{a} + \frac{\cos^3 ax}{3a}$

351 $\int \operatorname{sen}^4 ax\, dx = \frac{3x}{8} - \frac{\operatorname{sen} 2ax}{4a} + \frac{\operatorname{sen} 4ax}{32a}$

352 $\int \frac{dx}{\operatorname{sen}^2 ax} = -\frac{1}{a} \operatorname{cotg} ax$

353 $\int \frac{dx}{\operatorname{sen}^3 ax} = -\frac{\cos ax}{2a \operatorname{sen}^2 ax} + \frac{1}{2a} \ln \operatorname{tg} \frac{ax}{2}$

354 $\int \operatorname{sen} px \operatorname{sen} qx\, dx = \frac{\operatorname{sen}(p-q)x}{2(p-q)} - \frac{\operatorname{sen}(p+q)x}{2(p+q)}$ Si $p = \pm q$, véase 348 $\Leftrightarrow p \neq q$ o 0

355 $\int \frac{dx}{1 - \operatorname{sen} ax} = \frac{1}{a} \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

356 $\int \frac{x\, dx}{1 - \operatorname{sen} ax} = \frac{x}{a} \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{2}{a^2} \ln \operatorname{sen}\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

357 $\int \frac{dx}{1 + \operatorname{sen} ax} = -\frac{1}{a} \operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

358 $\int \frac{x\, dx}{1 + \operatorname{sen} ax} = -\frac{x}{a} \operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) + \frac{2}{a^2} \ln \operatorname{sen}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

359 $\int \frac{dx}{(1 - \operatorname{sen} ax)^2} = \frac{1}{2a} \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right) + \frac{1}{6a} \operatorname{tg}^3\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

360 $\int \frac{dx}{(1 + \operatorname{sen} ax)^2} = -\frac{1}{2a} \operatorname{tg}\left(\frac{\pi}{4} - \frac{ax}{2}\right) - \frac{1}{6a} \operatorname{tg}^3\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

361 $$\int \frac{dx}{p + q \operatorname{sen} ax} = \begin{cases} \dfrac{2}{a\sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \dfrac{p \operatorname{tg}\frac{ax}{2} + q}{\sqrt{p^2 - q^2}} \\[2ex] \dfrac{1}{a\sqrt{q^2 - p^2}} \ln\left(\dfrac{p \operatorname{tg}\frac{ax}{2} + q - \sqrt{q^2 - p^2}}{p \operatorname{tg}\frac{ax}{2} + q + \sqrt{q^2 - p^2}}\right) \end{cases}$$ Si $p = \pm q$, véanse 355 y 357

362 $\int \frac{dx}{(p + q \operatorname{sen} ax)^2} = \frac{q \cos ax}{a(p^2 - q^2)(p + q \operatorname{sen} ax)} + \frac{p}{(p^2 - q^2)} \int \frac{dx}{p + q \operatorname{sen} ax}$

Si $p = \pm q$, véanse 359 y 360

363 $\int \frac{dx}{p^2 + q^2 \operatorname{sen}^2 ax} = \frac{1}{a p \sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \frac{\sqrt{p^2 + q^2} \cdot \operatorname{tg} ax}{p}$

Ty → diferenciable hasta el orden n+1 en el ξ de (c h)
$\xi(c,h) \in S$

364 $$\int \frac{dx}{p^2 - q^2 \operatorname{sen}^2 ax} = \begin{cases} \dfrac{1}{a p \sqrt{p^2 - q^2}} \operatorname{tg}^{-1} \dfrac{\sqrt{p^2 - q^2} \cdot \operatorname{tg} ax}{p} \\[2ex] \dfrac{1}{2 a p \sqrt{q^2 - p^2}} \ln\left(\dfrac{\sqrt{q^2 - p^2} \cdot \operatorname{tg} ax + p}{\sqrt{q^2 - p^2} \cdot \operatorname{tg} ax - p}\right) \end{cases}$$

365 $\int x^m \operatorname{sen} ax\, dx = -\frac{x^m \cos ax}{a} + \frac{m x^{m-1} \operatorname{sen} ax}{a^2} - \frac{m(m-1)}{a^2} \int x^{m-2} \operatorname{sen} ax\, dx$

366 $\int \frac{\operatorname{sen} ax}{x^n}\, dx = -\frac{\operatorname{sen} ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\cos ax}{x^{n-1}}\, dx$ Véase 396

367 $\int \operatorname{sen}^n ax\, dx = -\frac{\operatorname{sen}^{n-1} ax \cos ax}{a n} + \frac{n-1}{n} \int \operatorname{sen}^{n-2} ax\, dx$

368 $\int \frac{dx}{\operatorname{sen}^n ax} = \frac{-\cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\operatorname{sen}^{n-2} ax}$

369 $\int \frac{x\, dx}{\operatorname{sen}^n ax} = \frac{-x \cos ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \frac{1}{a^2 (n-1)(n-2)\operatorname{sen}^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x\, dx}{\operatorname{sen}^{n-2} ax}$

INTEGRALES CON $\cos ax$

370 $\int \cos ax\, dx = \frac{\operatorname{sen} ax}{a}$

371 $\int x \cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x \operatorname{sen} ax}{a}$

372 $\int x^2 \cos ax\, dx = \frac{2x}{a^2} \cos ax + \left(\frac{x^2}{a} - \frac{2}{a^3}\right) \operatorname{sen} ax$

373 $\int x^3 \cos ax\, dx = \left(\frac{3x^2}{a^2} - \frac{6}{a^4}\right) \cos ax + \left(\frac{x^3}{a} - \frac{6x}{a^3}\right) \operatorname{sen} ax$

374 $\int \frac{\cos ax}{x}\, dx = \ln x - \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} - \frac{(ax)^6}{6 \cdot 6!} + \cdots = \ln x + \sum_{n=1}^{\infty} \frac{(-1)^n (ax)^{2n}}{(2n) \cdot (2n)!}$

375 $\int \frac{\cos ax}{x^2}\, dx = -\frac{\cos ax}{x} - a \int \frac{\operatorname{sen} ax}{x}\, dx$ Véase 374

376 $\int \frac{dx}{\cos ax} = \frac{1}{a} \ln(\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg}\left(\frac{\pi}{4} + \frac{ax}{2}\right)$

377 $\int \frac{x\, dx}{\cos ax} = \frac{1}{a^2}\left\{\frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5(ax)^6}{144} + \cdots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots\right\}$ E_n es nº de Euler

378 $\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\operatorname{sen} 2ax}{4a}$

379 $\int x \cos^2 ax\, dx = \frac{x^2}{4} + \frac{x \operatorname{sen} 2ax}{4a} + \frac{\cos 2ax}{8a^2}$

380 $\int \cos^3 ax\, dx = \frac{\operatorname{sen} ax}{a} - \frac{\operatorname{sen}^3 ax}{3a}$

381 $\int \cos^4 ax\, dx = \frac{3x}{8} + \frac{\text{sen}\, 2ax}{4\,a} + \frac{\text{sen}\, 4ax}{32\,a}$

382 $\int \frac{dx}{\cos^2 ax} = \frac{1}{a}\, \text{tg}\, ax$

383 $\int \frac{dx}{\cos^3 ax} = \frac{\text{sen}\, ax}{2\,a \cos^2 ax} + \frac{1}{2a} \ln \text{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right)$

384 $\int \cos px \cos qx\, dx = \frac{\text{sen}\,(p-q)x}{2\,(p-q)} + \frac{\text{sen}\,(p+q)x}{2\,(p+q)}$ Si $p = \pm q$, véase 378

385 $\int \frac{dx}{1 - \cos ax} = \frac{1}{a} \cot\text{g}\, \frac{ax}{2}$

386 $\int \frac{x\, dx}{1 - \cos ax} = -\frac{x}{a} \cot\text{g}\, \frac{ax}{2} + \frac{2}{a^2} \ln \text{sen}\, \frac{ax}{2}$

387 $\int \frac{dx}{1 + \cos ax} = \frac{1}{a}\, \text{tg}\, \frac{ax}{2}$

388 $\int \frac{x\, dx}{1 + \cos ax} = \frac{x}{a}\, \text{tg}\, \frac{ax}{2} + \frac{2}{a^2} \ln \cos \frac{ax}{2}$

389 $\int \frac{dx}{(1 - \cos ax)^2} = -\frac{1}{2a} \cot\text{g}\, \frac{ax}{2} - \frac{1}{6a} \cot\text{g}^3 \frac{ax}{2}$

390 $\int \frac{dx}{(1 + \cos ax)^2} = \frac{1}{2a}\, \text{tg}\, \frac{ax}{2} + \frac{1}{6a}\, \text{tg}^3 \frac{ax}{2}$

391 $\int \frac{dx}{p + q \cos ax} = \begin{cases} \frac{2}{a\sqrt{p^2 - q^2}}\, \text{tg}^{-1} \sqrt{\frac{p-q}{p+q}} \cdot \text{tg}\, \frac{ax}{2} \\ \frac{1}{a\sqrt{q^2 - p^2}} \ln \left(\frac{\text{tg}\, \frac{ax}{2} + \sqrt{\frac{q+p}{q-p}}}{\text{tg}\, \frac{ax}{2} - \sqrt{\frac{q+p}{q-p}}} \right) \end{cases}$ Si $p = \pm q$, véanse 385 y 387

392 $\int \frac{dx}{(p + q \cos ax)^2} = \frac{\text{sen}\, ax}{a\,(q^2 - p^2)(p + q \cos ax)} - \frac{p}{(q^2 - p^2)} \int \frac{dx}{p + q \cos ax}$

Si $p = \pm q$, véanse 389 y 390

393 $\int \frac{dx}{p^2 + q^2 \cos^2 ax} = \frac{1}{a\,p\sqrt{p^2 + q^2}}\, \text{tg}^{-1} \frac{p\, \text{tg}\, ax}{\sqrt{p^2 + q^2}}$

394 $\int \frac{dx}{p^2 - q^2 \cos^2 ax} = \begin{cases} \frac{1}{a\,p\sqrt{p^2 - q^2}}\, \text{tg}^{-1} \frac{p\, \text{tg}\, ax}{\sqrt{p^2 + q^2}} \\ \frac{1}{2\,a\,p\sqrt{q^2 - p^2}} \ln \left(\frac{p\, \text{tg}\, ax - \sqrt{q^2 - p^2}}{p\, \text{tg}\, ax + \sqrt{q^2 - p^2}} \right) \end{cases}$

395 $\int x^m \cos ax\, dx = \frac{x^m\, \text{sen}\, ax}{a} + \frac{m\, x^{m-1} \cos ax}{a^2} - \frac{m\,(m-1)}{a^2} \int x^{m-2} \cos ax\, dx$

396 $\int \frac{\cos ax}{x^n}\, dx = -\frac{\cos ax}{(n-1)\, x^{n-1}} - \frac{a}{n-1} \int \frac{\text{sen}\, ax}{x^{n-1}}\, dx$ Véase 366

397 $\int \cos^n ax\, dx = \frac{\cos^{n-1} ax\, \text{sen}\, ax}{a\,n} + \frac{n-1}{n} \int \cos^{n-2} ax\, dx$

398 $\int \frac{dx}{\cos^n ax} = \frac{\text{sen}\, ax}{a\,(n-1) \cos^{n-1} ax} + \frac{(n-2)}{(n-1)} \int \frac{dx}{\cos^{n-2} ax}$

399 $\int \frac{x\, dx}{\cos^n ax} = \frac{x\, \text{sen}\, ax}{a\,(n-1) \cos^{n-1} ax} - \frac{1}{a^2\,(n-1)(n-2) \cos^{n-2} ax} + \frac{(n-2)}{(n-1)} \int \frac{x \cdot dx}{\cos^{n-2} ax}$

INTEGRALES CON sen ax y cos ax

400 $\int \text{sen}\, ax \cos ax\, dx = \frac{\text{sen}^2\, ax}{2\,a}$

401 $\int \text{sen}\, px \cos qx\, dx = -\frac{\cos(p-q)x}{2\,(p-q)} - \frac{\cos(p+q)x}{2\,(p+q)}$

402 $\int \text{sen}^n\, ax \cos ax\, dx = \frac{\text{sen}^{n+1}\, ax}{(n+1)\,a}$ Si $n = -1$, véase 441

403 $\int \text{sen}\, ax \cos^n ax\, dx = -\frac{\cos^{n+1} ax}{(n+1)\,a}$ Si $n = -1$, véase 430

404 $\int \text{sen}^2\, ax \cos^2 ax\, dx = \frac{x}{8} - \frac{\text{sen}\, 4ax}{32\,a}$

405 $\int \frac{dx}{\text{sen}\, ax \cos ax} = \frac{1}{a} \ln \text{tg}\, ax$

406 $\int \frac{dx}{\text{sen}^2\, ax \cos ax} = \frac{1}{a} \ln \text{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right) - \frac{1}{a\, \text{sen}\, ax}$

407 $\int \frac{dx}{\text{sen}\, ax \cos^2 ax} = \frac{1}{a} \ln \text{tg}\, \frac{ax}{2} + \frac{1}{a \cos ax}$

408 $\int \frac{dx}{\text{sen}^2\, ax \cos^2 ax} = -\frac{2 \cot\text{g}\, 2ax}{a}$

409 $\int \frac{\text{sen}^2\, ax}{\cos ax}\, dx = \frac{1}{a} \ln \text{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right) - \frac{\text{sen}\, ax}{a}$

410 $\int \frac{\cos^2 ax}{\text{sen}\, ax}\, dx = \frac{1}{a} \ln \text{tg}\, \frac{ax}{2} + \frac{\cos ax}{a}$

411 $\int \frac{dx}{(1 \pm \text{sen}\, ax) \cos ax} = \mp \frac{1}{2\,a\,(1 \pm \text{sen}\, ax)} + \frac{1}{2\,a} \ln \text{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right)$

412 $\int \frac{dx}{(1 \pm \cos ax)\, \text{sen}\, ax} = \pm \frac{1}{2\,a\,(1 \pm \cos ax)} + \frac{1}{2\,a} \ln \text{tg}\, \frac{ax}{2}$

413 $\int \frac{dx}{\text{sen}\, ax \pm \cos ax} = \frac{1}{a\sqrt{2}} \ln \text{tg} \left(\pm \frac{\pi}{8} + \frac{ax}{2} \right)$

414 $\int \frac{\text{sen}\, ax\, dx}{\text{sen}\, ax \pm \cos ax} = \frac{x}{2} \mp \frac{1}{2\,a} \ln (\text{sen}\, ax \pm \cos ax)$

415 $$\int \frac{\cos ax\,dx}{\operatorname{sen} ax \pm \cos ax} = \pm\frac{x}{2} + \frac{1}{2a}\ln(\operatorname{sen} ax \pm \cos ax)$$

416 $$\int \frac{\operatorname{sen} ax\,dx}{p + q\cos ax} = -\frac{1}{aq}\ln(p + q\cos ax)$$

417 $$\int \frac{\cos ax\,dx}{p + q\operatorname{sen} ax} = \frac{1}{aq}\ln(p + q\operatorname{sen} ax)$$

418 $$\int \frac{\operatorname{sen} ax\,dx}{(p + q\cos ax)^n} = \frac{1}{aq(n-1)(p + q\cos ax)^{n-1}}$$

419 $$\int \frac{\cos ax\,dx}{(p + q\operatorname{sen} ax)^n} = \frac{-1}{aq(n-1)(p + q\operatorname{sen} ax)^{n-1}}$$

420 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax} = \frac{1}{a\sqrt{p^2+q^2}}\ln\operatorname{tg}\left(\frac{ax + \operatorname{tg}^{-1}(q/p)}{2}\right)$$

421 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax + r} = \begin{cases} \dfrac{2}{a\sqrt{r^2-p^2-q^2}}\operatorname{tg}^{-1}\left(\dfrac{p + (r-q)\operatorname{tg}(ax/2)}{\sqrt{r^2-p^2-q^2}}\right) \\[2ex] \dfrac{1}{a\sqrt{p^2+q^2-r^2}}\ln\left(\dfrac{p - \sqrt{p^2+q^2-r^2} + (r-q)\operatorname{tg}(ax/2)}{p + \sqrt{p^2+q^2-r^2} + (r-q)\operatorname{tg}(ax/2)}\right) \end{cases}$$

Si $r = q$ véase 422. Si $r^2 = p^2 + q^2$ véase 423.

422 $$\int \frac{dx}{p\operatorname{sen} ax + q(1 + \cos ax)} = \frac{1}{ap}\ln\left(q + p\operatorname{tg}\frac{ax}{2}\right)$$

423 $$\int \frac{dx}{p\operatorname{sen} ax + q\cos ax \pm \sqrt{p^2+q^2}} = \frac{-1}{a\sqrt{p^2+q^2}}\operatorname{tg}\left(\frac{\pi}{4} \mp \frac{ax + \operatorname{tg}^{-1}(q/p)}{2}\right)$$

424 $$\int \frac{dx}{p^2\operatorname{sen}^2 ax + q^2\cos^2 ax} = \frac{1}{apq}\operatorname{tg}^{-1}\left(\frac{p\operatorname{tg} ax}{q}\right)$$

425 $$\int \frac{dx}{p^2\operatorname{sen}^2 ax - q^2\cos^2 ax} = \frac{1}{2apq}\ln\left(\frac{p\operatorname{tg} ax - q}{p\operatorname{tg} ax + q}\right)$$

426 $$\int \operatorname{sen}^m ax\cos^n ax\,dx = \begin{cases} -\dfrac{\operatorname{sen}^{m-1} ax\cos^{n+1} ax}{a(m+n)} + \dfrac{(m-1)}{(m+n)}\displaystyle\int \operatorname{sen}^{m-2} ax\cos^n ax\,dx \\[2ex] \dfrac{\operatorname{sen}^{m+1} ax\cos^{n-1} ax}{a(m+n)} + \dfrac{(n-1)}{(m+n)}\displaystyle\int \operatorname{sen}^m ax\cos^{n-2} ax\,dx \end{cases}$$

427 $$\int \frac{\operatorname{sen}^m ax}{\cos^n ax}\,dx = \begin{cases} \dfrac{\operatorname{sen}^{m-1} ax}{a(n-1)\cos^{n-1} ax} - \dfrac{m-1}{n-1}\displaystyle\int \dfrac{\operatorname{sen}^{m-2} ax}{\cos^{n-2} ax}\,dx \\[2ex] \dfrac{\operatorname{sen}^{m+1} ax}{a(n-1)\cos^{n-1} ax} - \dfrac{m-n+2}{n-1}\displaystyle\int \dfrac{\operatorname{sen}^m ax}{\cos^{n-2} ax}\,dx \\[2ex] \dfrac{-\operatorname{sen}^{m-1} ax}{a(m-n)\cos^{n-1} ax} + \dfrac{m-1}{m-n}\displaystyle\int \dfrac{\operatorname{sen}^{m-2} ax}{\cos^n ax}\,dx \end{cases}$$

428 $$\int \frac{\cos^m ax}{\operatorname{sen}^n ax}\,dx = \begin{cases} \dfrac{-\cos^{m-1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-1}{n-1}\displaystyle\int \dfrac{\cos^{m-2} ax}{\operatorname{sen}^{n-2} ax}\,dx \\[2ex] \dfrac{-\cos^{m+1} ax}{a(n-1)\operatorname{sen}^{n-1} ax} - \dfrac{m-n+2}{n-1}\displaystyle\int \dfrac{\cos^m ax}{\operatorname{sen}^{n-2} ax}\,dx \\[2ex] \dfrac{\cos^{m-1} ax}{a(m-n)\operatorname{sen}^{n-1} ax} + \dfrac{m-1}{m-n}\displaystyle\int \dfrac{\cos^{m-2} ax}{\operatorname{sen}^n ax}\,dx \end{cases}$$

429 $$\int \operatorname{sen}^m ax\cos^n ax\,dx = \begin{cases} \dfrac{1}{a(n-1)\operatorname{sen}^{m-1} ax\cos^{n-1} ax} + \dfrac{m+n-2}{n-1}\displaystyle\int \dfrac{dx}{\operatorname{sen}^m ax\cos^{n-2} ax} \\[2ex] \dfrac{-1}{a(m-1)\operatorname{sen}^{m-1} ax\cos^{n-1} ax} + \dfrac{m+n-2}{m-1}\displaystyle\int \dfrac{dx}{\operatorname{sen}^{m-2} ax\cos^n ax} \end{cases}$$

INTEGRALES CON tg ax

430 $$\int \operatorname{tg} ax\,dx = -\frac{1}{a}\ln\cos ax = \frac{1}{a}\ln\sec ax$$

431 $$\int \operatorname{tg}^2 ax\,dx = \frac{\operatorname{tg} ax}{a} - x$$

432 $$\int \operatorname{tg}^3 ax\,dx = \frac{\operatorname{tg}^2 ax}{2a} + \frac{1}{a}\ln\cos ax$$

433 $$\int \operatorname{tg}^n ax\sec^2 ax\,dx = \frac{\operatorname{tg}^{n+1} ax}{(n+1)a}$$

434 $$\int \frac{\sec^2 ax}{\operatorname{tg} ax}\,dx = \frac{1}{a}\ln\operatorname{tg} ax$$

435 $$\int \frac{dx}{\operatorname{tg} ax} = \frac{1}{a}\ln\operatorname{sen} ax$$

436 $$\int x\operatorname{tg} ax\,dx = \frac{1}{a^2}\left\{\frac{(ax)^3}{3} + \frac{(ax)^5}{15} + \frac{2(ax)^7}{105} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n+1}}{(2n+1)!} + \cdots\right\}$$

B_n es nº de Bernoulli tanto en 436 como en 437

437 $$\int \frac{\operatorname{tg} ax}{x}\,dx = ax + \frac{(ax)^3}{9} + \frac{2(ax)^5}{75} + \cdots + \frac{2^{2n}(2^{2n}-1)B_n(ax)^{2n-1}}{(2n-1)(2n)!} + \cdots$$

438 $$\int x\operatorname{tg}^2 ax\,dx = \frac{x\operatorname{tg} ax}{a} + \frac{1}{a^2}\ln\cos ax - \frac{x^2}{2}$$

439 $$\int \frac{dx}{p + q\operatorname{tg} ax} = \frac{px}{p^2+q^2} + \frac{q}{a(p^2+q^2)}\ln(q\operatorname{sen} ax + p\cos ax)$$

440 $$\int \operatorname{tg}^n ax\,dx = \frac{\operatorname{tg}^{n-1} ax}{(n-1)a} - \int \operatorname{tg}^{n-2} ax\,dx$$

INTEGRALES CON cotg ax

441 $\int \cot g\, ax\, dx = \frac{1}{a} \ln \operatorname{sen} ax$

442 $\int \cot g^2 ax\, dx = - \frac{\cot g\, ax}{a} - x$

443 $\int \cot g^3 ax\, dx = - \frac{\cot g^2 ax}{2\, a} - \frac{1}{a} \ln \operatorname{sen} ax$

444 $\int \cot g^n ax \operatorname{cosec}^2 ax\, dx = - \frac{\cot g^{n+1} ax}{(n+1)\, a}$

445 $\int \frac{\operatorname{cosec}^2 ax}{\cot g\, ax}\, dx = - \frac{1}{a} \ln \cot g\, ax$

446 $\int \frac{dx}{\cot g\, ax} = - \frac{1}{a} \ln \cos ax$

447 $\int x \cot g\, ax\, dx = \frac{1}{a^2} \left[ax - \frac{(ax)^3}{9} - \frac{(ax)^5}{225} - \dots - \frac{2^{2n} B_n (ax)^{2n+1}}{(2n+1)!} - \dots \right]$

B_n es nº de Bernoulli tanto en 447 como en 448

448 $\int \frac{\cot g\, ax}{x}\, dx = - \frac{1}{ax} - \frac{ax}{3} - \frac{(ax)^3}{135} - \dots - \frac{2^{2n} B_n (ax)^{2n-1}}{(2n-1)(2n)!} - \dots$

449 $\int x \cot g^2 ax\, dx = \frac{x \cot g\, ax}{a} + \frac{1}{a^2} \ln \operatorname{sen} ax - \frac{x^2}{2}$

450 $\int \frac{dx}{p + q \cot g\, ax} = \frac{px}{p^2 + q^2} - \frac{q}{a(p^2+q^2)} \ln (q \operatorname{sen} ax + p \cos ax)$

451 $\int \cot g^n ax\, dx = - \frac{\cot g^{n-1} ax}{(n-1)\, a} - \int \cot g^{n-2} ax\, dx$

INTEGRALES CON sec ax

452 $\int \sec ax\, dx = \frac{1}{a} \ln (\sec ax + \operatorname{tg} ax) = \frac{1}{a} \ln \operatorname{tg} \left(\frac{\pi}{4} + \frac{ax}{2} \right)$

453 $\int \sec^2 ax\, dx = \frac{\operatorname{tg} ax}{a}$

454 $\int \sec^3 ax\, dx = \frac{\sec ax \operatorname{tg} ax}{2\, a} + \frac{1}{2\, a} \ln (\sec ax + \operatorname{tg} ax)$

455 $\int \sec^n ax \operatorname{tg} ax\, dx = \frac{\sec^n ax}{n\, a}$

456 $\int \frac{dx}{\sec ax} = \frac{\operatorname{sen} ax}{a}$

457 $\int x \sec ax\, dx = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} + \frac{(ax)^4}{8} + \frac{5\,(ax)^6}{144} + \dots + \frac{E_n (ax)^{2n+2}}{(2n+2)(2n)!} + \dots \right\}$ E_n es nº de Euler

458 $\int \frac{\sec ax}{x}\, dx = \ln x + \frac{(ax)^2}{4} + \frac{5\,(ax)^4}{96} + \frac{61\,(ax)^6}{4320} + \dots + \frac{E_n (ax)^{2n}}{2n\,(2n)!} + \dots$

459 $\int x \sec^2 ax\, dx = \frac{x}{a} \operatorname{tg} ax + \frac{1}{a^2} \ln \cos ax$

460 $\int \frac{dx}{q + p \sec ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q + p \cos ax}$ Véase 391

461 $\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \operatorname{tg} ax}{a\,(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx$

INTEGRALES CON cosec ax

462 $\int \operatorname{cosec} ax\, dx = \frac{1}{a} \ln (\operatorname{cosec} ax - \cot g\, ax) = \frac{1}{a} \ln \operatorname{tg} \frac{ax}{2}$

463 $\int \operatorname{cosec}^2 ax\, dx = - \frac{\cot g\, ax}{a}$

464 $\int \operatorname{cosec}^3 ax\, dx = - \frac{\operatorname{cosec} ax \cot g\, ax}{2\, a} + \frac{1}{2\, a} \ln \operatorname{tg} \frac{ax}{2}$

465 $\int \operatorname{cosec}^n ax \cot g\, ax\, dx = - \frac{\operatorname{cosec}^n ax}{n\, a}$

466 $\int \frac{dx}{\operatorname{cosec} x} = - \frac{\cos ax}{a}$

467 $\int x \operatorname{cosec} ax\, dx = \frac{1}{a^2} \left\{ ax + \frac{(ax)^3}{18} + \frac{7\,(ax)^5}{1800} + \dots + \frac{2\,(2^{2n-1} - 1)\, B_n (ax)^{2n+1}}{(2n+1)!} + \dots \right\}$

B_n es nº de Bernoulli

468 $\int \frac{\operatorname{cosec} ax}{x}\, dx = - \frac{1}{ax} + \frac{ax}{6} + \frac{7\,(ax)^3}{1080} + \dots + \frac{2\,(2^{2n-1} - 1)\, B_n (ax)^{2n-1}}{(2n-1)(2n)!} + \dots \}$

469 $\int x \operatorname{cosec}^2 ax\, dx = - \frac{x}{a} \cot g\, ax + \frac{1}{a^2} \ln \operatorname{sen} ax$

470 $\int \frac{dx}{q + p \operatorname{cosec} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{q + p \operatorname{sen} ax}$ Véase 361

471 $\int \operatorname{cosec}^n ax\, dx = - \frac{\operatorname{cosec}^{n-2} ax \cot g\, ax}{a\,(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} ax\, dx$

INTEGRALES DE FUNCIONES TRIGONOMETRICAS INVERSAS

472 $\int \operatorname{sen}^{-1} \frac{x}{a}\, dx = x \operatorname{sen}^{-1} \frac{x}{a} + \sqrt{a^2 - x^2}$

473 $\int x \operatorname{sen}^{-1} \frac{x}{a}\, dx = \left(\frac{x^2}{2} - \frac{a^2}{4} \right) \operatorname{sen}^{-1} \frac{x}{a} + \frac{x \sqrt{a^2 - x^2}}{4}$

474 $\int x^2 \operatorname{sen}^{-1} \frac{x}{a}\, dx = \frac{x^3}{3} \operatorname{sen}^{-1} \frac{x}{a} + \frac{(x^2 + 2\, a^2) \sqrt{a^2 - x^2}}{9}$

475 $\int x^m \operatorname{sen}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\operatorname{sen}^{-1}\frac{x}{a} - \frac{1}{m+1}\int \frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

476 $\int \frac{\operatorname{sen}^{-1}\frac{x}{a}}{x}\,dx = \frac{x}{a} + \frac{(x/a)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3\,(x/a)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5\,(x/a)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$

477 $\int \frac{\operatorname{sen}^{-1}\frac{x}{a}}{x^2}\,dx = -\frac{\operatorname{sen}^{-1}\frac{x}{a}}{x} - \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

478 $\int \left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 dx = x\left(\operatorname{sen}^{-1}\frac{x}{a}\right)^2 - 2x + 2\sqrt{a^2-x^2}\,\operatorname{sen}^{-1}\frac{x}{a}$

479 $\int \cos^{-1}\frac{x}{a}\,dx = x\cos^{-1}\frac{x}{a} - \sqrt{a^2-x^2}$

480 $\int x\cos^{-1}\frac{x}{a}\,dx = \left(\frac{x^2}{2}-\frac{a^2}{4}\right)\cos^{-1}\frac{x}{a} - \frac{x\sqrt{a^2-x^2}}{4}$

481 $\int x^2\cos^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}\cos^{-1}\frac{x}{a} - \frac{(x^2+2a^2)\sqrt{a^2-x^2}}{9}$

482 $\int x^m\cos^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\cos^{-1}\frac{x}{a} + \frac{1}{m+1}\int\frac{x^{m+1}}{\sqrt{a^2-x^2}}\,dx$

483 $\int \frac{\cos^{-1}\frac{x}{a}}{x}\,dx = \frac{\pi}{2}\ln x - \int \frac{\operatorname{sen}^{-1}\frac{x}{a}}{x}\,dx$ Véase 476

484 $\int \frac{\cos^{-1}\frac{x}{a}}{x^2}\,dx = -\frac{\cos^{-1}(x/a)}{x} + \frac{1}{a}\ln\left(\frac{a+\sqrt{a^2-x^2}}{x}\right)$

485 $\int \left(\cos^{-1}\frac{x}{a}\right)^2 dx = x\left(\cos^{-1}\frac{x}{a}\right)^2 - 2x - 2\sqrt{a^2-x^2}\,\cos^{-1}\frac{x}{a}$

486 $\int tg^{-1}\frac{x}{a}\,dx = x\,tg^{-1}\frac{x}{a} - \frac{a}{2}\ln(x^2+a^2)$

487 $\int x\,tg^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\,tg^{-1}\frac{x}{a} - \frac{ax}{2}$

488 $\int x^2\,tg^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}tg^{-1}\frac{x}{a} - \frac{ax^2}{6} + \frac{a^3}{6}\ln(x^2+a^2)$

489 $\int x^m\,tg^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}tg^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^{m+1}}{x^2+a^2}\,dx$

490 $\int \frac{tg^{-1}(x/a)}{x}\,dx = \frac{x}{a} - \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} - \frac{(x/a)^7}{7^2} + \cdots$

491 $\int \frac{tg^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}tg^{-1}\frac{x}{a} - \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

492 $\int cotg^{-1}\frac{x}{a}\,dx = x\,cotg^{-1}\frac{x}{a} + \frac{a}{2}\ln(x^2+a^2)$

493 $\int x\,cotg^{-1}\frac{x}{a}\,dx = \frac{1}{2}(x^2+a^2)\,cotg^{-1}\frac{x}{a} + \frac{ax}{2}$

494 $\int x^2\,cotg^{-1}\frac{x}{a}\,dx = \frac{x^3}{3}cotg^{-1}\frac{x}{a} + \frac{ax^2}{6} - \frac{a^3}{6}\ln(x^2+a^2)$

495 $\int x^m\,cotg^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}cotg^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^{m+1}}{x^2+a^2}\,dx$

496 $\int \frac{cotg^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x - \int\frac{tg^{-1}\frac{x}{a}}{x}\,dx$ Véase 490

497 $\int \frac{cotg^{-1}(x/a)}{x^2}\,dx = -\frac{1}{x}cotg^{-1}\frac{x}{a} + \frac{1}{2a}\ln\left(\frac{x^2+a^2}{x^2}\right)$

498 $\int \sec^{-1}\frac{x}{a}\,dx = \begin{cases} x\sec^{-1}\frac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right); & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ x\sec^{-1}\frac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right); & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

499 $\int x\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\sec^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^2}{2}\sec^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2}; & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

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500 $\int x^2\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^3}{3}\sec^{-1}\frac{x}{a} - \frac{a\,x\sqrt{x^2-a^2}}{2} - \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right); & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^3}{3}\sec^{-1}\frac{x}{a} + \frac{a\,x\sqrt{x^2-a^2}}{2} + \frac{a^3}{6}\ln\left(x+\sqrt{x^2-a^2}\right); & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

501 $\int x^m\sec^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\sec^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^{m+1}}{m+1}\sec^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}}; & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

502 $\int \frac{\sec^{-1}(x/a)}{x}\,dx = \frac{\pi}{2}\ln x + \frac{a}{x} + \frac{(a/x)^3}{2\cdot 3\cdot 3} + \frac{1\cdot 3\,(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \frac{1\cdot 3\cdot 5\,(a/x)^7}{2\cdot 4\cdot 6\cdot 7\cdot 7} + \cdots$

503 $\int \frac{\sec^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\sec^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{a\,x}; & 0<\sec^{-1}\frac{x}{a}<\frac{\pi}{2} \\ -\frac{\sec^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{a\,x}; & \frac{\pi}{2}<\sec^{-1}\frac{x}{a}<\pi \end{cases}$

504 $\int \operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} x\operatorname{cosec}^{-1}\frac{x}{a} + a\ln\left(x+\sqrt{x^2-a^2}\right); & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ x\operatorname{cosec}^{-1}\frac{x}{a} - a\ln\left(x+\sqrt{x^2-a^2}\right); & -\frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<0 \end{cases}$

505 $\int x\operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^2}{2}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{a\sqrt{x^2-a^2}}{2}; & 0<\operatorname{cosec}^{-1}\frac{x}{a}<\frac{\pi}{2} \\ \frac{x^2}{2}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{a\sqrt{x^2-a^2}}{2}; & -\frac{\pi}{2}<\operatorname{cosec}^{-1}\frac{x}{a}<0 \end{cases}$

506 $$\int x^2 \operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x}{3}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{ax\sqrt{x-a}}{2} + \frac{a}{6}\ln(x+\sqrt{x^2-a^2});\ 0 < \operatorname{cosec}^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^3}{3}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{ax\sqrt{x^2-a^2}}{2} - \frac{a^3}{6}\ln(x+\sqrt{x^2-a^2});\ -\frac{\pi}{2} < \operatorname{cosec}^{-1}\frac{x}{a} < 0 \end{cases}$$

507 $$\int x^m \operatorname{cosec}^{-1}\frac{x}{a}\,dx = \begin{cases} \frac{x^{m+1}}{m+1}\operatorname{cosec}^{-1}\frac{x}{a} + \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}};\ 0 < \operatorname{cosec}^{-1}\frac{x}{a} < \frac{\pi}{2} \\ \frac{x^{m+1}}{m+1}\operatorname{cosec}^{-1}\frac{x}{a} - \frac{a}{m+1}\int\frac{x^m\,dx}{\sqrt{x^2-a^2}};\ \frac{\pi}{2} < \operatorname{cosec}^{-1}\frac{x}{a} < \pi \end{cases}$$

508 $$\int\frac{\operatorname{cosec}^{-1}(x/a)}{x}\,dx = -\left\{\frac{a}{x} + \frac{(a/x)^3}{2\cdot3\cdot3} + \frac{1\cdot3\,(a/x)^5}{2\cdot4\cdot5\cdot5} + \frac{1\cdot3\cdot5\,(a/x)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots\right\}$$

509 $$\int\frac{\operatorname{cosec}^{-1}(x/a)}{x^2}\,dx = \begin{cases} -\frac{\operatorname{cosec}^{-1}(x/a)}{x} - \frac{\sqrt{x^2-a^2}}{a\,x};\ 0 < \operatorname{cosec}^{-1}\frac{x}{a} < \frac{\pi}{2} \\ -\frac{\operatorname{cosec}^{-1}(x/a)}{x} + \frac{\sqrt{x^2-a^2}}{a\,x};\ -\frac{\pi}{2} < \sec^{-1}\frac{x}{a} < 0 \end{cases}$$

INTEGRALES CON e^{ax}

510 $$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

511 $$\int x\,e^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

512 $$\int x^2 e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$$

513 $$\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx \left(= \sum_{k=1}^{n}\frac{(-1)^k\,k!}{a^k} \quad \text{Si } n \text{ es natural}\right)$$

514 $$\int\frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot1!} + \frac{(ax)^2}{2\cdot2!} + \frac{(ax)^3}{3\cdot3!} + \cdots$$

515 $$\int\frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)\,x^{n-1}} + \frac{a}{n-1}\int\frac{e^{ax}}{x^{n-1}}\,dx$$

516 $$\int\frac{dx}{p+q\,e^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p+q\,e^{ax})$$

517 $$\int\frac{dx}{(p+q\,e^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap\,(p+q\,e^{ax})} - \frac{1}{ap^2}\ln(p+q\,e^{ax})$$

518 $$\int\frac{dx}{p\,e^{ax}+q\,e^{-ax}} = \begin{cases} \frac{1}{a\sqrt{pq}}\,tg^{-1}\left(\sqrt{\frac{p}{q}}\,e^{ax}\right) \\ \frac{1}{2a\sqrt{-pq}}\ln\left(\frac{e^{ax}-\sqrt{-q/p}}{e^{ax}+\sqrt{-q/p}}\right) \end{cases}$$

519 $$\int e^{ax}\operatorname{sen}bx\,dx = \frac{e^{ax}(a\operatorname{sen}bx - b\cos bx)}{a^2+b^2}$$ más

520 $$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\operatorname{sen}bx)}{a^2+b^2}$$

521 $$\int x\,e^{ax}\operatorname{sen}bx\,dx = \frac{x\,e^{ax}(a\operatorname{sen}bx - b\cos bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\operatorname{sen}bx - 2ab\cos bx\}}{(a^2+b^2)^2}$$

522 $$\int x\,e^{ax}\cos bx\,dx = \frac{x\,e^{ax}(a\cos bx + b\operatorname{sen}bx)}{a^2+b^2} - \frac{e^{ax}\{(a^2-b^2)\cos bx + 2ab\operatorname{sen}bx\}}{(a^2+b^2)^2}$$

523 $$\int e^{ax}\ln x\,dx = \frac{e^{ax}\ln x}{a} - \frac{1}{a}\int\frac{e^{ax}}{x}\,dx$$

524 $$\int e^{ax}\operatorname{sen}^n bx\,dx = \frac{e^{ax}\operatorname{sen}^{n-1}bx\,(a\operatorname{sen}bx - nb\cos bx)}{a^2+n^2b^2} + \frac{n(n-1)\,b^2}{a^2+n^2b^2}\int e^{ax}\operatorname{sen}^{n-2}bx\,dx$$

525 $$\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1}bx\,(a\cos bx + nb\operatorname{sen}bx)}{a^2+n^2b^2} + \frac{n(n-1)\,b^2}{a^2+n^2b^2}\int e^{ax}\cos^{n-2}bx\,dx$$

INTEGRALES CON $\ln x$

526 $$\int \ln x\,dx = x\ln x - x$$

527 $$\int x\ln x\,dx = \frac{x^2}{2}\left\{\ln x - \frac{1}{2}\right\}$$

528 $$\int x^m\ln x\,dx = \frac{x^{m+1}}{m+1}\left\{\ln x - \frac{1}{m+1}\right\}$$ Si $m = -1$ véase 529

529 $$\int\frac{\ln x}{x}\,dx = \frac{1}{2}\ln^2 x$$

530 $$\int\frac{\ln x}{x^2}\,dx = -\frac{\ln x}{x} - \frac{1}{x}$$

531 $$\int\ln^2 x\,dx = x\ln^2 x - 2x\ln x + 2x$$

532 $$\int\frac{\ln^n x}{x}\,dx = \frac{\ln^{n+1}x}{n+1}$$ Si $n = -1$ véase 533

533 $$\int\frac{dx}{x\ln x} = \ln(\ln x)$$

534 $$\int\frac{dx}{\ln x} = \ln(\ln x) + \ln x + \frac{\ln^2 x}{2\cdot2!} + \frac{\ln^3 x}{3\cdot3!} + \cdots$$

535 $$\int\frac{x^m\,dx}{\ln x} = \ln(\ln x) + (m+1)\ln x + \frac{(m+1)^2\ln^2 x}{2\cdot2!} + \frac{(m+1)^3\ln^3 x}{3\cdot3!} + \cdots$$

536 $$\int\ln^n x\,dx = x\ln^n x - n\int\ln^{n-1}x\,dx$$

537 $$\int x^m\ln^n x\,dx = \frac{\ln^n x}{m+1} - \frac{n}{m+1}\int x^m\ln^{n-1}x\,dx$$ Si $m = -1$ véase 532

538 $$\int\ln(x^2+a^2)\,dx = x\ln(x^2+a^2) - 2x + 2a\,tg^{-1}\frac{x}{a}$$

539 $$\int\ln(x^2-a^2)\,dx = x\ln(x^2-a^2) - 2x + a\ln\left(\frac{x+a}{x-a}\right)$$

540 $$\int x^m\ln(x^2\pm a^2)\,dx = \frac{x^{m+1}\ln(x^2\pm a^2)}{m+1} - \frac{2}{m+1}\int\frac{x^{m+2}}{(x^2\pm a^2)}\,dx$$

INTEGRALES CON senh ax

541 $\displaystyle\int \operatorname{senh} ax\, dx = \frac{\cosh ax}{a}$

542 $\displaystyle\int x \operatorname{senh} ax\, dx = -\frac{\operatorname{senh} ax}{a^2} + \frac{x \cosh ax}{a}$

543 $\displaystyle\int x^2 \operatorname{senh} ax\, dx = -\frac{2x}{a^2} \operatorname{senh} ax + \left(\frac{2}{a^3} + \frac{x^2}{a}\right) \cosh ax$

544 $\displaystyle\int \frac{\operatorname{senh} ax}{x}\, dx = ax + \frac{(ax)^3}{3 \cdot 3!} + \frac{(ax)^5}{5 \cdot 5!} - \cdots = \sum_{n=1}^{\infty} \frac{(ax)^{2n-1}}{(2n-1)(2n-1)!}$

545 $\displaystyle\int \frac{\operatorname{senh} ax}{x^2}\, dx = -\frac{\operatorname{senh} ax}{x} + a \int \frac{\cosh ax}{x}\, dx$ Véase 566

546 $\displaystyle\int \frac{dx}{\operatorname{senh} ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2}$

547 $\displaystyle\int \frac{x\, dx}{\operatorname{senh} ax} = \frac{1}{a^2} \left\{ ax - \frac{(ax)^3}{18} + \frac{7\,(ax)^5}{1800} + \cdots + \frac{2\,(-1)^n\,(2^{2n-1} - 1)\, B_n\, (ax)^{2n+1}}{(2n+1)!} + \cdots \right\}$

B_n es nº de Bernoulli

548 $\displaystyle\int \operatorname{senh}^2 ax\, dx = -\frac{x}{2} + \frac{\operatorname{senh} ax \cosh ax}{2a}$

549 $\displaystyle\int x \operatorname{senh}^2 ax\, dx = -\frac{x^2}{4} + \frac{x \operatorname{senh} 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$

550 $\displaystyle\int \frac{dx}{\operatorname{senh}^2 ax} = -\frac{1}{a} \operatorname{cotgh} ax$

551 $\displaystyle\int \operatorname{senh} px \operatorname{senh} qx\, dx = -\frac{\operatorname{senh}(p-q)x}{2(p-q)} + \frac{\operatorname{senh}(p+q)x}{2(p+q)}$ Si $p = \pm q$, véase 548

552 $\displaystyle\int \operatorname{senh} px \operatorname{sen} qx\, dx = \frac{p \cosh px \operatorname{sen} qx - q \operatorname{senh} px \cos qx}{p^2 + q^2}$

553 $\displaystyle\int \operatorname{senh} px \cos qx\, dx = \frac{p \cosh px \cos qx + q \operatorname{senh} px \operatorname{sen} qx}{p^2 + q^2}$

554 $\displaystyle\int \frac{dx}{p + q \operatorname{senh} ax} = \frac{1}{a\sqrt{p^2+q^2}} \ln \left(\frac{q\, e^{ax} + p - \sqrt{p^2+q^2}}{q\, e^{ax} + p + \sqrt{p^2+q^2}} \right)$

555 $\displaystyle\int \frac{dx}{(p + q \operatorname{senh} ax)^2} = \frac{-q \cosh ax}{a(p^2+q^2)(p + q \operatorname{senh} ax)} + \frac{p}{(p^2+q^2)} \int \frac{dx}{p + q \operatorname{senh} ax}$

556 $\displaystyle\int \frac{dx}{p^2 + q^2 \operatorname{senh}^2 ax} = \begin{cases} \dfrac{1}{a p \sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \dfrac{\sqrt{q^2 - p^2} \cdot \operatorname{tgh} ax}{p} \\[2ex] \dfrac{1}{2 a p \sqrt{p^2 - q^2}} \ln \left(\dfrac{p + \sqrt{p^2 - q^2} \cdot \operatorname{tgh} ax}{p - \sqrt{p^2 - q^2} \cdot \operatorname{tgh} ax} \right) \end{cases}$

557 $\displaystyle\int \frac{dx}{p^2 - q^2 \operatorname{senh}^2 ax} = \frac{1}{2 a p \sqrt{p^2 + q^2}} \ln \left(\frac{p + \sqrt{p^2 + q^2} \cdot \operatorname{tgh} ax}{p - \sqrt{p^2 + q^2} \cdot \operatorname{tgh} ax} \right)$

558 $\displaystyle\int x^m \operatorname{senh} ax\, dx = \frac{x^m \cosh ax}{a} - \frac{m}{a} \int x^{m-1} \cosh ax\, dx$ Véase 586

559 $\displaystyle\int \frac{\operatorname{senh} ax}{x^n}\, dx = -\frac{\operatorname{senh} ax}{(n-1)\, x^{n-1}} + \frac{a}{n-1} \int \frac{\cosh ax}{x^{n-1}}\, dx$ Véase 587

560 $\displaystyle\int \operatorname{senh}^n ax\, dx = \frac{\operatorname{senh}^{n-1} ax \cosh ax}{a\, n} - \frac{n-1}{n} \int \operatorname{senh}^{n-2} ax\, dx$

561 $\displaystyle\int \frac{dx}{\operatorname{senh}^n ax} = \frac{-\cosh ax}{a(n-1) \operatorname{senh}^{n-1} ax} - \frac{n-2}{n-1} \int \frac{dx}{\operatorname{senh}^{n-2} ax}$

562 $\displaystyle\int \frac{x\, dx}{\operatorname{senh}^n ax} = \frac{-x \cosh ax}{a(n-1) \operatorname{senh}^{n-1} ax} - \frac{1}{a^2 (n-1)(n-2) \operatorname{senh}^{n-2} ax} - \frac{n-2}{n-1} \int \frac{x\, dx}{\operatorname{senh}^{n-2} ax}$

INTEGRALES CON cosh ax

563 $\displaystyle\int \cosh ax\, dx = \frac{\operatorname{senh} ax}{a}$

564 $\displaystyle\int x \cosh ax\, dx = -\frac{\cosh ax}{a^2} + \frac{x \operatorname{senh} ax}{a}$

565 $\displaystyle\int x^2 \cosh ax\, dx = -\frac{2x}{a^2} \cosh ax + \left(\frac{x^2}{a} + \frac{2}{a^3}\right) \operatorname{senh} ax$

566 $\displaystyle\int \frac{\cosh ax}{x}\, dx = \ln x + \frac{(ax)^2}{2 \cdot 2!} + \frac{(ax)^4}{4 \cdot 4!} + \frac{(ax)^6}{6 \cdot 6!} + \cdots = \ln x + \sum_{n=1}^{\infty} \frac{(ax)^{2n}}{(2n) \cdot (2n)!}$

567 $\displaystyle\int \frac{\cosh ax}{x^2}\, dx = -\frac{\cosh ax}{x} + a \int \frac{\operatorname{senh} ax}{x}\, dx$ Véase 544

568 $\displaystyle\int \frac{dx}{\cosh ax} = \frac{2}{a} \operatorname{tg}^{-1} e^{ax}$

569 $\displaystyle\int \frac{x\, dx}{\cosh ax} = \frac{1}{a^2} \left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5\,(ax)^6}{144} - \cdots + \frac{(-1)^n\, E_n\, (ax)^{2n+2}}{(2n+2)(2n)!} + \cdots \right\}$ E_n es nº de Euler

570 $\displaystyle\int \cosh^2 ax\, dx = \frac{x}{2} + \frac{\operatorname{senh} ax \cosh ax}{2}$

571 $\displaystyle\int x \cosh^2 ax\, dx = \frac{x^2}{4} + \frac{x \operatorname{senh} 2ax}{4a} - \frac{\cosh 2ax}{8a^2}$

572 $\displaystyle\int \frac{dx}{\cosh^2 ax} = \frac{1}{a} \operatorname{tgh} ax$

573 $\displaystyle\int \cosh px \cosh qx\, dx = \frac{\operatorname{senh}(p-q)x}{2(p-q)} + \frac{\operatorname{senh}(p+q)x}{2(p+q)}$ Si $p = \pm q$, véase 570

574 $\displaystyle\int \cosh px \operatorname{sen} qx\, dx = \frac{p \operatorname{senh} px \operatorname{sen} qx - q \cosh px \cos qx}{p^2 + q^2}$

575 $\displaystyle\int \cosh px \cos qx\, dx = \frac{p \operatorname{senh} px \cos qx + q \cosh px \operatorname{sen} qx}{p^2 + q^2}$

576 $$\int \frac{dx}{1 - \cosh ax} = \frac{1}{a} \operatorname{cotgh} \frac{ax}{2}$$

577 $$\int \frac{x\,dx}{1 - \cosh ax} = \frac{x}{a} \operatorname{cotgh} \frac{ax}{2} - \frac{2}{a^2} \ln \operatorname{senh} \frac{ax}{2}$$

578 $$\int \frac{dx}{1 + \cosh ax} = \frac{1}{a} \operatorname{tgh} \frac{ax}{2}$$

579 $$\int \frac{x\,dx}{1 + \cosh ax} = \frac{x}{a} \operatorname{tgh} \frac{ax}{2} - \frac{2}{a^2} \ln \cosh \frac{ax}{2}$$

580 $$\int \frac{dx}{(1 - \cosh ax)^2} = \frac{1}{2a} \operatorname{cotgh} \frac{ax}{2} - \frac{1}{6a} \operatorname{cotgh}^3 \frac{ax}{2}$$

581 $$\int \frac{dx}{(1 + \cosh ax)^2} = \frac{1}{2a} \operatorname{tgh} \frac{ax}{2} - \frac{1}{6a} \operatorname{tgh}^3 \frac{ax}{2}$$

582 $$\int \frac{dx}{p + q \cosh ax} = \begin{cases} \dfrac{2}{a\sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \dfrac{p + q e^{ax}}{\sqrt{q^2 - p^2}} \\[2ex] \dfrac{1}{a\sqrt{p^2 - q^2}} \ln\left(\dfrac{q e^{ax} + p - \sqrt{p^2 - q^2}}{q e^{ax} + p + \sqrt{p^2 - q^2}} \right) \end{cases}$$

583 $$\int \frac{dx}{(p + q \cosh ax)^2} = \frac{q \operatorname{senh} ax}{a (q^2 - p^2)(p + q \cosh ax)} - \frac{p}{(q^2 - p^2)} \int \frac{dx}{p + q \cosh ax}$$

584 $$\int \frac{dx}{p^2 + q^2 \cosh^2 ax} = \begin{cases} \dfrac{1}{2ap\sqrt{p^2 + q^2}} \ln\left(\dfrac{p \operatorname{tgh} ax + \sqrt{p^2 + q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 + q^2}} \right) \\[2ex] \dfrac{1}{ap\sqrt{p^2 + q^2}} \operatorname{tg}^{-1} \dfrac{p \operatorname{tgh} ax}{\sqrt{p^2 + q^2}} \end{cases}$$

585 $$\int \frac{dx}{p^2 - q^2 \cosh^2 ax} = \begin{cases} \dfrac{-1}{ap\sqrt{q^2 - p^2}} \operatorname{tg}^{-1} \dfrac{p \operatorname{tgh} ax}{\sqrt{q^2 - p^2}} \\[2ex] \dfrac{1}{2ap\sqrt{p^2 - q^2}} \ln\left(\dfrac{p \operatorname{tgh} ax + \sqrt{p^2 - q^2}}{p \operatorname{tgh} ax - \sqrt{p^2 - q^2}} \right) \end{cases}$$

586 $$\int x^m \cosh ax\,dx = \frac{x^m \operatorname{senh} ax}{a} - \frac{m}{a} \int x^{m-1} \operatorname{senh} ax\,dx$$ Véase 558

587 $$\int \frac{\cosh ax}{x^n}\,dx = -\frac{\cosh ax}{(n-1)x^{n-1}} + \frac{a}{n-1} \int \frac{\operatorname{senh} ax}{x^{n-1}}\,dx$$ Véase 559

588 $$\int \cosh^n ax\,dx = \frac{\cosh^{n-1} ax \operatorname{senh} ax}{an} + \frac{n-1}{n} \int \cosh^{n-2} ax\,dx$$

589 $$\int \frac{dx}{\cosh^n ax} = \frac{\operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{n-2}{n-1} \int \frac{dx}{\cosh^{n-2} ax}$$

590 $$\int \frac{x\,dx}{\cosh^n ax} = \frac{x \operatorname{senh} ax}{a(n-1)\cosh^{n-1} ax} + \frac{1}{a^2 (n-1)(n-2)\cosh^{n-2} ax} + \frac{n-2}{n-1} \int \frac{x\,dx}{\cosh^{n-2} ax}$$

INTEGRALES CON senh ax y cosh ax

591 $$\int \operatorname{senh} ax \cosh ax\,dx = \frac{\operatorname{senh}^2 ax}{2a}$$

592 $$\int \operatorname{senh} px \cosh qx\,dx = \frac{\cosh (p-q)x}{2(p-q)} + \frac{\cosh (p+q)x}{2(p+q)}$$

593 $$\int \operatorname{senh}^n ax \cosh ax\,dx = \frac{\operatorname{senh}^{n+1} ax}{(n+1)a}$$ Si $n = -1$, véase 615

594 $$\int \operatorname{senh} ax \cosh^n ax\,dx = \frac{\cosh^{n+1} ax}{(n+1)a}$$ Si $n = -1$, véase 604

595 $$\int \operatorname{senh}^2 ax \cosh^2 ax\,dx = -\frac{x}{8} + \frac{\operatorname{senh} 4ax}{32a}$$

596 $$\int \frac{dx}{\operatorname{senh} ax \cosh ax} = \frac{1}{a} \ln \operatorname{tgh} ax$$

597 $$\int \frac{dx}{\operatorname{senh}^2 ax \cosh ax} = -\frac{1}{a} \operatorname{tg}^{-1} \operatorname{senh} ax - \frac{\cosh ax}{a}$$

598 $$\int \frac{dx}{\operatorname{senh} ax \cosh^2 ax} = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\operatorname{sech} ax}{a}$$

599 $$\int \frac{dx}{\operatorname{senh}^2 ax \cosh^2 ax} = -\frac{2 \operatorname{cotgh} 2ax}{a}$$

600 $$\int \frac{\operatorname{senh}^2 ax}{\cosh ax}\,dx = -\frac{1}{a} \operatorname{tg}^{-1} \operatorname{senh} ax + \frac{\operatorname{senh} ax}{a}$$

601 $$\int \frac{\cosh^2 ax}{\operatorname{senh} ax}\,dx = \frac{1}{a} \ln \operatorname{tgh} \frac{ax}{2} + \frac{\cosh ax}{a}$$

602 $$\int \frac{dx}{(1 + \operatorname{senh} ax)\cosh ax} = \frac{1}{2a} \ln\left(\frac{1 + \operatorname{senh} ax}{\cosh ax} \right) + \frac{1}{a} \operatorname{tg}^{-1} e^{ax}$$

603 $$\int \frac{dx}{(\cosh ax \pm 1)\operatorname{senh} ax} = \pm \frac{1}{2a(\cosh ax \pm 1)} \pm \frac{1}{2a} \ln \operatorname{tgh} \frac{ax}{2}$$

INTEGRALES CON tgh ax

604 $$\int \operatorname{tgh} ax\,dx = \frac{1}{a} \ln \cosh ax$$

605 $$\int \operatorname{tgh}^2 ax\,dx = -\frac{\operatorname{tgh} ax}{a} + x$$

606 $$\int \operatorname{tgh}^3 ax\,dx = -\frac{\operatorname{tgh}^2 ax}{2a} + \frac{1}{a} \ln \cosh ax$$

607 $\int \mathrm{tgh}^n\, ax\ \mathrm{sech}^2\, ax\, dx = \frac{\mathrm{tgh}^{n+1}\, ax}{(n+1)\, a}$

608 $\int \frac{\mathrm{sech}^2\, ax}{\mathrm{tgh}\, ax}\, dx = \frac{1}{a} \ln \mathrm{tgh}\, ax$

609 $\int \frac{dx}{\mathrm{tgh}\, ax} = \frac{1}{a} \ln \mathrm{senh}\, ax$

610 $\int x\, \mathrm{tgh}\, ax\, dx = \frac{1}{a^2}\left(\frac{(ax)^3}{3} - \frac{(ax)^5}{15} + \frac{2\,(ax)^7}{105} - \dots + \frac{(-1)^{n-1}\, 2^{2n}\, (2^{2n}-1)\, B_n\, (ax)^{2n+1}}{(2n+1)!} + \dots \right)$

B_n es nº de Bernoulli tanto en 610 como en 611

611 $\int \frac{\mathrm{tgh}\, ax}{x}\, dx = ax - \frac{(ax)^3}{9} + \frac{2\,(ax)^5}{75} - \dots + \frac{(-1)^{n-1}\, 2^{2n}\, (2^{2n}-1)\, B_n\, (ax)^{2n-1}}{(2n-1)\,(2n)!} + \dots$

612 $\int x\, \mathrm{tgh}^2\, ax\, dx = -\frac{x\, \mathrm{tgh}\, ax}{a} + \frac{1}{a^2} \ln \cosh ax + \frac{x^2}{2}$

613 $\int \frac{dx}{p + q\, \mathrm{tgh}\, ax} = \frac{px}{p^2 - q^2} - \frac{q}{a\,(p^2 - q^2)} \ln (q\, \mathrm{senh}\, ax + p \cosh ax)$

614 $\int \mathrm{tgh}^n\, ax\, dx = -\frac{\mathrm{tgh}^{n-1}\, ax}{(n-1)\, a} + \int \mathrm{tgh}^{n-2}\, ax\, dx$

INTEGRALES CON cotgh ax

615 $\int \mathrm{cotgh}\, ax\, dx = \frac{1}{a} \ln \mathrm{senh}\, ax$

616 $\int \mathrm{cotgh}^2\, ax\, dx = -\frac{\mathrm{cotgh}\, ax}{a} + x$

617 $\int \mathrm{cotgh}^3\, ax\, dx = -\frac{\mathrm{cotgh}^2\, ax}{2\, a} + \frac{1}{a} \ln \mathrm{senh}\, ax$

618 $\int \mathrm{cotgh}^n\, ax\ \mathrm{cosech}^2\, ax\, dx = -\frac{\mathrm{cotgh}^{n+1}\, ax}{(n+1)\, a}$

619 $\int \frac{\mathrm{cosech}^2\, ax}{\mathrm{cotgh}\, ax}\, dx = -\frac{1}{a} \ln \mathrm{cotgh}\, ax$

620 $\int \frac{dx}{\mathrm{cotgh}\, ax} = \frac{1}{a} \ln \cosh ax$

621 $\int x\, \mathrm{cotgh}\, ax\, dx = \frac{1}{a^2}\left(ax + \frac{(ax)^3}{9} - \frac{(ax)^5}{225} + \dots + \frac{(-1)^{n-1}\, 2^{2n}\, B_n\, (ax)^{2n+1}}{(2n+1)!} + \dots \right)$

B_n es nº de Bernoulli tanto en 621 como en 622

622 $\int \frac{\mathrm{cotgh}\, ax}{x}\, dx = -\frac{1}{ax} + \frac{ax}{3} - \frac{(ax)^3}{135} + \dots + \frac{(-1)^n\, 2^{2n}\, B_n\, (ax)^{2n-1}}{(2n-1)\,(2n)!} + \dots$

623 $\int x\, \mathrm{cotgh}^2\, ax\, dx = -\frac{x\, \mathrm{cotgh}\, ax}{a} + \frac{1}{a^2} \ln \mathrm{senh}\, ax + \frac{x^2}{2}$

624 $\int \frac{dx}{p + q\, \mathrm{cotgh}\, ax} = \frac{px}{p^2 - q^2} - \frac{q}{a\,(p^2 - q^2)} \ln (p\, \mathrm{senh}\, ax + q \cosh ax)$

625 $\int \mathrm{cotgh}^n\, ax\, dx = -\frac{\mathrm{cotgh}^{n-1}\, ax}{(n-1)\, a} + \int \mathrm{cotgh}^{n-1}\, ax\, dx$

INTEGRALES CON sech ax

626 $\int \mathrm{sech}\, ax\, dx = \frac{2}{a}\, \mathrm{tg}^{-1}\, e^{ax}$

627 $\int \mathrm{sech}^2\, ax\, dx = \frac{\mathrm{tgh}\, ax}{a}$

628 $\int \mathrm{sech}^3\, ax\, dx = \frac{\mathrm{sech}\, ax\ \mathrm{tgh}\, ax}{2\, a} + \frac{1}{2\, a}\, \mathrm{tg}^{-1}\, \mathrm{senh}\, ax$

629 $\int \mathrm{sech}^n\, ax\ \mathrm{tgh}\, ax\, dx = -\frac{\mathrm{sech}^n\, ax}{n\, a}$

630 $\int \frac{dx}{\mathrm{sech}\, ax} = \frac{\mathrm{senh}\, ax}{a}$

631 $\int x\, \mathrm{sech}\, ax\, dx = \frac{1}{a^2}\left\{ \frac{(ax)^2}{2} - \frac{(ax)^4}{8} + \frac{5\,(ax)^6}{144} - \dots + \frac{(-1)^n\, E_n\, (ax)^{2n+2}}{(2n+2)\,(2n)!} + \dots \right\}$ E_n es nº de Euler

632 $\int \frac{\mathrm{sech}\, ax}{x}\, dx = \ln x - \frac{(ax)^2}{4} + \frac{5\,(ax)^4}{96} - \frac{61\,(ax)^6}{4320} + \dots + \frac{(-1)^n\, E_n\, (ax)^{2n}}{2n\,(2n)!} + \dots$

633 $\int x\, \mathrm{sech}^2\, ax\, dx = \frac{x}{a}\, \mathrm{tgh}\, ax - \frac{1}{a^2} \ln \cosh ax$

634 $\int \frac{dx}{q + p\, \mathrm{sech}\, ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \cosh ax}$ Véase 582

635 $\int \mathrm{sech}^n\, ax\, dx = \frac{\mathrm{sech}^{n-2}\, ax\ \mathrm{tgh}\, ax}{a\,(n-1)} + \frac{n-2}{n-1} \int \mathrm{sech}^{n-2}\, ax\, dx$

INTEGRALES CON cosech ax

636 $\int \mathrm{cosech}\, ax\, dx = \frac{1}{a} \ln \mathrm{tgh}\, \frac{ax}{2}$

637 $\int \mathrm{cosech}^2\, ax\, dx = -\frac{\mathrm{cotgh}\, ax}{a}$

638 $\int \mathrm{cosech}^3\, ax\, dx = -\frac{\mathrm{cosech}\, ax\ \mathrm{cotgh}\, ax}{2\, a} - \frac{1}{2\, a} \ln \mathrm{tgh}\, \frac{ax}{2}$

639 $\int \mathrm{cosech}^n\, ax\ \mathrm{cotgh}\, ax\, dx = -\frac{\mathrm{cosech}^n\, ax}{n\, a}$

640 $\int \frac{dx}{\mathrm{cosech}\, x} = \frac{\cosh ax}{a}$

641 $\int x \operatorname{cosech} ax\, dx = \frac{1}{a^2}\left\{ax - \frac{(ax)^3}{18} + \frac{7\,(ax)^5}{1800} - \dots + \frac{(-1)^n\, 2\,(2^{2n-1} - 1)\, B_n\, (ax)^{2n+1}}{(2n+1)!} + \dots\right\}$

B_n es nº de Bernoulli

642 $\int \frac{\operatorname{cosech} ax}{x}\, dx = -\frac{1}{ax} - \frac{ax}{6} + \frac{7\,(ax)^3}{1080} - \dots + \frac{(-1)^n\, 2\,(2^{2n-1} - 1)\, B_n\, (ax)^{2n-1}}{(2n-1)\,(2n)!} + \dots$

643 $\int x \operatorname{cosech}^2 ax\, dx = -\frac{x}{a} \operatorname{cotgh} ax + \frac{1}{a^2} \ln \operatorname{senh} ax$

644 $\int \frac{dx}{q + p \operatorname{cosech} ax} = \frac{x}{q} - \frac{p}{q} \int \frac{dx}{p + q \operatorname{senh} ax}$ Véase 554

645 $\int \operatorname{cosech}^n ax\, dx = -\frac{\operatorname{cosech}^{n-2} ax \operatorname{cotgh} ax}{a\,(n-1)} + \frac{n-2}{n-1} \int \operatorname{cosech}^{n-2} ax\, dx$

INTEGRALES DE FUNCIONES HIPERBOLICAS INVERSAS

646 $\int \operatorname{senh}^{-1} \frac{x}{a}\, dx = x \operatorname{senh}^{-1} \frac{x}{a} - \sqrt{a^2 + x^2}$

647 $\int x \operatorname{senh}^{-1} \frac{x}{a}\, dx = \left(\frac{x^2}{2} + \frac{a^2}{4}\right) \operatorname{senh}^{-1} \frac{x}{a} - \frac{x\sqrt{a^2 + x^2}}{4}$

648 $\int x^2 \operatorname{senh}^{-1} \frac{x}{a}\, dx = \frac{x^3}{3} \operatorname{senh}^{-1} \frac{x}{a} + \frac{(-x^2 + 2a^2)\sqrt{a^2 + x^2}}{9}$

649 $\int x^m \operatorname{sen}^{-1} \frac{x}{a}\, dx = \frac{x^{m+1}}{m+1} \operatorname{senh}^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{a^2 + x^2}}\, dx$

650 $$\int \frac{\operatorname{senh}^{-1} \frac{x}{a}}{x}\, dx = \begin{cases} \frac{x}{a} - \frac{(x/a)^3}{2\cdot3\cdot3} + \frac{1\cdot3\,(x/a)^5}{2\cdot4\cdot5\cdot5} - \frac{1\cdot3\cdot5\,(x/a)^7}{2\cdot4\cdot6\cdot7\cdot7} + \cdots & |x| < a \\ \frac{\ln^2 (2x/a)}{2} - \frac{(a/x)^2}{2\cdot2\cdot2} + \frac{1\cdot3\,(a/x)^4}{2\cdot4\cdot4\cdot4} - \frac{1\cdot3\cdot5\,(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} + \cdots & x > a \\ \frac{-\ln^2 \left(\frac{-2x}{a}\right)}{2} + \frac{(a/x)^2}{2\cdot2\cdot2} - \frac{1\cdot3\,(a/x)^4}{2\cdot4\cdot4\cdot4} + \frac{1\cdot3\cdot5\,(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} - \cdots & x < -a \end{cases}$$

651 $\int \frac{\operatorname{senh}^{-1} \frac{x}{a}}{x^2}\, dx = -\frac{\operatorname{senh}^{-1} \frac{x}{a}}{x} - \frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$

652 $$\int \cosh^{-1} \frac{x}{a}\, dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} > 0 \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

653 $$\int x \cosh^{-1} \frac{x}{a}\, dx = \begin{cases} \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \cos^{-1} \frac{x}{a} - \frac{x\sqrt{x^2 - a^2}}{4}; & \cosh^{-1} \frac{x}{a} > 0 \\ \left(\frac{x^2}{2} - \frac{a^2}{4}\right) \cos^{-1} \frac{x}{a} - \frac{x\sqrt{x^2 - a^2}}{4}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

654 $$\int x^2 \cosh^{-1} \frac{x}{a}\, dx = \begin{cases} \frac{x}{3} \cosh^{-1} \frac{x}{a} - \frac{x + a \sqrt{x - a}}{9}; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^3}{3} \cosh^{-1} \frac{x}{a} + \frac{(x^2 + 2a^2)\sqrt{x^2 - a^2}}{9}; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

655 $$\int x^m \cosh^{-1} \frac{x}{a}\, dx = \begin{cases} \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} - \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}}\, dx; & \cosh^{-1} \frac{x}{a} > 0 \\ \frac{x^{m+1}}{m+1} \cosh^{-1} \frac{x}{a} + \frac{1}{m+1} \int \frac{x^{m+1}}{\sqrt{x^2 - a^2}}\, dx; & \cosh^{-1} \frac{x}{a} < 0 \end{cases}$$

656 $\int \frac{\cosh^{-1} \frac{x}{a}}{x}\, dx = \pm\left(\frac{\ln^2 (2x/a)}{2} + \frac{(a/x)^2}{2\cdot2\cdot2} + \frac{1\cdot3\,(a/x)^4}{2\cdot4\cdot4\cdot4} + \frac{1\cdot3\cdot5\,(a/x)^6}{2\cdot4\cdot6\cdot6\cdot6} + \cdots\right)$

$+$ si $\cosh^{-1} \frac{x}{a} > 0$; $-$ si $\cosh^{-1} \frac{x}{a} < 0$.

657 $\int \frac{\cosh^{-1} \frac{x}{a}}{x^2}\, dx = -\frac{\cosh^{-1} (x/a)}{x} \mp \frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 + x^2}}{x}\right)$

$-$ si $\cosh^{-1} \frac{x}{a} > 0$; $+$ si $\cosh^{-1} \frac{x}{a} < 0$.

658 $\int \operatorname{tgh}^{-1} \frac{x}{a}\, dx = x \operatorname{tgh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(a^2 - x^2)$

659 $\int x \operatorname{tgh}^{-1} \frac{x}{a}\, dx = \frac{1}{2}(x^2 - a^2) \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax}{2}$

660 $\int x^2 \operatorname{tgh}^{-1} \frac{x}{a}\, dx = \frac{x^3}{3} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{ax^2}{6} + \frac{a^3}{6} \ln(a^2 - x^2)$

661 $\int x^m \operatorname{tgh}^{-1} \frac{x}{a}\, dx = \frac{x^{m+1}}{m+1} \operatorname{tgh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 - x^2}\, dx$

662 $\int \frac{\operatorname{tgh}^{-1} (x/a)}{x}\, dx = \frac{x}{a} + \frac{(x/a)^3}{3^2} + \frac{(x/a)^5}{5^2} + \frac{(x/a)^7}{7^2} + \cdots$

663 $\int \frac{\operatorname{tgh}^{-1} (x/a)}{x^2}\, dx = -\frac{1}{x} \operatorname{tgh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln\left(\frac{x^2}{a^2 - x^2}\right)$

664 $\int \operatorname{cotgh}^{-1} \frac{x}{a}\, dx = x \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{a}{2} \ln(x^2 - a^2)$

665 $\int x \operatorname{cotgh}^{-1} \frac{x}{a}\, dx = \frac{1}{2}(x^2 - a^2) \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{ax}{2}$

666 $\int x^2 \operatorname{cotgh}^{-1} \frac{x}{a}\, dx = \frac{ax^2}{6} + \frac{x^3}{3} \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{a^3}{6} \ln(x^2 - a^2)$

667 $\int x^m \operatorname{cotgh}^{-1} \frac{x}{a}\, dx = \frac{x^{m+1}}{m+1} \operatorname{cotgh}^{-1} \frac{x}{a} - \frac{a}{m+1} \int \frac{x^{m+1}}{a^2 - x^2}\, dx$

668 $\int \frac{\operatorname{cotgh}^{-1} (x/a)}{x}\, dx = -\left(\frac{a}{x} + \frac{(a/x)^3}{3^2} + \frac{(a/x)^5}{5^2} + \frac{(a/x)^7}{7^2} + \cdots\right)$

669 $\int \frac{\operatorname{cotgh}^{-1} (x/a)}{x^2}\, dx = -\frac{1}{x} \operatorname{cotgh}^{-1} \frac{x}{a} + \frac{1}{2a} \ln\left[\frac{x^2}{x^2 - a^2}\right]$

670 $$\int \operatorname{sech}^{-1} \frac{x}{a}\, dx = \begin{cases} x \operatorname{sech}^{-1} \frac{x}{a} + a \operatorname{sen}^{-1} (x/a); & \sec^{-1} \frac{x}{a} > 0 \\ x \operatorname{sech}^{-1} \frac{x}{a} - a \operatorname{sen}^{-1} (x/a); & \sec^{-1} \frac{x}{a} < 0 \end{cases}$$

671 $$\int x \,\mathrm{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x}{2}\,\mathrm{sech}^{-1}\dfrac{x}{a} - \dfrac{a\sqrt{a^2 - x^2}}{2}; & \sec^{-1}\dfrac{x}{a} > 0 \\ \dfrac{x^2}{2}\,\mathrm{sech}^{-1}\dfrac{x}{a} + \dfrac{a\sqrt{a^2 - x^2}}{2}; & \sec^{-1}\dfrac{x}{a} < 0 \end{cases}$$

672 $$\int x^m \,\mathrm{sech}^{-1}\frac{x}{a}\,dx = \begin{cases} \dfrac{x^{m+1}}{m+1}\,\mathrm{sech}^{-1}\dfrac{x}{a} + \dfrac{a}{m+1}\displaystyle\int \dfrac{x^m\,dx}{\sqrt{a^2 - x^2}}; & \mathrm{sech}^{-1}\dfrac{x}{a} > 0 \\ \dfrac{x^{m+1}}{m+1}\,\mathrm{sech}^{-1}\dfrac{x}{a} - \dfrac{a}{m+1}\displaystyle\int \dfrac{x^m\,dx}{\sqrt{a^2 - x^2}}; & \mathrm{sech}^{-1}\dfrac{x}{a} < 0 \end{cases}$$

673 $$\int \frac{\mathrm{sech}^{-1}(x/a)}{x}\,dx = \mp\left(\frac{\ln(x/a)\ln(4x/a)}{2} + \frac{(x/a)^2}{2\cdot 2\cdot 2} + \frac{1\cdot 3\,(x/a)^4}{2\cdot 4\cdot 4\cdot 4} + \frac{1\cdot 3\cdot 5\,(x/a)^6}{2\cdot 4\cdot 6\cdot 6\cdot 6} + \cdots\right)$$

$-$ si $\mathrm{sech}^{-1}\frac{x}{a} > 0$; $+$ si $\mathrm{sech}^{-1}\frac{x}{a} < 0$.

674 $$\int \mathrm{cosech}^{-1}\frac{x}{a}\,dx = x\,\mathrm{cosech}^{-1}\frac{x}{a} \pm a\,\mathrm{senh}^{-1}\frac{x}{a};\ [+ \text{ si } x > 0,\ - \text{ si } x < 0]$$

675 $$\int x\,\mathrm{cosech}^{-1}\frac{x}{a}\,dx = \frac{x^2}{2}\,\mathrm{cosech}^{-1}\frac{x}{a} \pm \frac{a\sqrt{x^2 + a^2}}{2};\ [+ \text{ si } x > 0,\ - \text{ si } x < 0]$$

676 $$\int x^m\,\mathrm{cosech}^{-1}\frac{x}{a}\,dx = \frac{x^{m+1}}{m+1}\,\mathrm{cosech}^{-1}\frac{x}{a} \pm \frac{a}{m+1}\int \frac{x^m\,dx}{\sqrt{x^2 + a^2}};\ [+ \text{ si } x > 0,\ - \text{ si } x < 0]$$

677 $$\int \frac{\mathrm{cosec}^{-1}(x/a)}{x}\,dx = \begin{cases} \dfrac{\ln(x/a)\ln(4x/a)}{2} + \dfrac{(x/a)^2}{2\cdot 2\cdot 2} - \dfrac{1\cdot 3\,(x/a)^4}{2\cdot 4\cdot 4\cdot 4} + \cdots; & 0 < x < a \\ \dfrac{\ln(-x/a)\ln(-x/4a)}{2} - \dfrac{(x/a)^2}{2\cdot 2\cdot 2} + \dfrac{1\cdot 3\,(x/a)^4}{2\cdot 4\cdot 4\cdot 4} - \cdots; & -a < x < 0 \\ -\dfrac{a}{x} + \dfrac{(a/x)^3}{2\cdot 3\cdot 3} - \dfrac{1\cdot 3\,(a/x)^5}{2\cdot 4\cdot 5\cdot 5} + \cdots; & |x| > a \end{cases}$$

INTEGRALES DEFINIDAS

PROPIEDADES

678 $$\int_a^b [k\,f(x) + r\,g(x) - n\,h(x)]\,dx = k\int_a^b f(x)\,dx + r\int_a^b g(x)\,dx - n\int_a^b h(x)\,dx;\ a,b,k,r,n \in \mathbb{R}$$

679 $$\int_a^a f(x)\,dx = 0$$

680 $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

681 $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

682 $$\int_a^b f(x)\,dx = f(c)\,(b - a)$$ Para algún c tal que $a < c < b$, f continua en $[a;b]$

683 Si $f'(x) = F(x) \Rightarrow \displaystyle\int_a^b f(x)\,dx = F(b) - F(a)$ (Regla de Barrow)

INTEGRALES IMPROPIAS

684 $$\int_a^{+\infty} f(x)\,dx = \lim_{z\to+\infty}\int_a^z f(x)\,dx$$

685 $$\int_{-\infty}^{+\infty} f(x)\,dx = \lim_{\substack{b\to+\infty\\ a\to-\infty}}\int_a^b f(x)\,dx$$

686 $$\int_a^b f(x)\,dx = \lim_{\theta\to 0}\int_a^{b-\theta} f(x)\,dx$$ Si b es punto singular de f, $\theta > 0$

687 $$\int_a^b f(x)\,dx = \lim_{\theta\to 0}\int_{a+\theta}^{b} f(x)\,dx$$ Si a es punto singular de f, $\theta > 0$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES TRIGONOMETRICAS

688 $$\int_0^{\pi} \mathrm{sen}\,nx\ \mathrm{sen}\,kx\,dx = \int_0^{\pi}\cos nx\,\cos kx\,dx = \begin{cases} 0 & \text{si } n \neq k \\ \dfrac{\pi}{2} & \text{si } n = k \end{cases};\ n, k \in \mathbb{Z}$$

689 $$\int_0^{\pi} \mathrm{sen}\,kx\,\cos nx\,dx = \begin{cases} 0; & \text{si } n + k \text{ es impar} \\ \dfrac{2k}{k^2 - n^2}; & \text{si } n + k \text{ es par} \end{cases};\ n, k \in \mathbb{Z}$$

690 $$\int_0^{\frac{\pi}{2}} \mathrm{sen}^2 x\,dx = \int_0^{\frac{\pi}{2}} \cos^2 x\,dx = \frac{\pi}{4}$$

691 $$\int_0^{\frac{\pi}{2}} \mathrm{sen}^{2k} x\,dx = \int_0^{\frac{\pi}{2}} \cos^{2k} x\,dx = \frac{1\cdot 3\cdot 5\cdots(2k-1)}{2\cdot 4\cdot 6\cdots 2k}\cdot\frac{\pi}{2} \qquad k \in \mathbb{N}$$

692 $$\int_0^{\frac{\pi}{2}} \mathrm{sen}^{2k+1} x\,dx = \int_0^{\frac{\pi}{2}} \cos^{2k+1} x\,dx = \frac{2\cdot 4\cdot 6\cdots 2k}{1\cdot 3\cdot 5\cdots(2k+1)} \qquad k \in \mathbb{N}$$

693 $$\int_0^{\frac{\pi}{2}} \mathrm{sen}^{2p-1} x\,\cos^{2q-1} x\,dx = \frac{\Gamma(p)\cdot\Gamma(q)}{2\cdot\Gamma(p+q)}$$ Γ: Función Gamma (Ver apéndice)

694 $$\int_0^{\infty} \frac{\mathrm{sen}\,kx}{x}\,dx = \begin{cases} \pi/2 & k > 0 \\ 0 & k = 0 \\ -\pi/2 & k < 0 \end{cases}$$

595 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} px \cos qx}{x}\, dx = \begin{cases} \pi/2; & 0 < p < q \\ 0; & 0 < q < p \\ \pi/4; & p = q > 0 \end{cases}$

696 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} px \cos qx}{x^2}\, dx = \begin{cases} p\,\pi/2; & 0 < p \leq q \\ q\,\pi/2; & 0 < q \leq p \end{cases}$

697 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen}^2 px}{x^2}\, dx = \frac{p\,\pi}{2}$

698 $\displaystyle\int_0^{\infty} \frac{1 - \cos px}{x^2}\, dx = \frac{p\,\pi}{2}$

699 $\displaystyle\int_0^{\infty} \frac{\cos px - \cos qx}{x}\, dx = \ln \frac{q}{p}$

700 $\displaystyle\int_0^{\infty} \frac{\cos px - \cos qx}{x^2}\, dx = \frac{(q - p)\,\pi}{2}$

701 $\displaystyle\int_0^{\infty} \frac{\cos nx}{x^2 + a^2}\, dx = \frac{\pi}{2\,a}\, e^{-na}$

702 $\displaystyle\int_0^{\infty} \frac{x \operatorname{sen} nx}{x^2 + a^2}\, dx = \frac{\pi}{2}\, e^{-na}$

703 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} nx}{x\,(x^2 + a^2)}\, dx = \frac{\pi}{2\,a^2}\,(1 - e^{-na})$

704 $\displaystyle\int_0^{2\pi} \frac{dx}{a + b \operatorname{sen} x} = \frac{2\,\pi}{\sqrt{a^2 - b^2}}$

705 $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{a + b \cos x} = \frac{\cos^{-1}(b/a)}{\sqrt{a^2 - b^2}}$

706 $\displaystyle\int_0^{2\pi} \frac{dx}{(a + b \operatorname{sen} x)^2} = \int_0^{2\pi} \frac{dx}{(a + b \cos x)^2} = \frac{2\,\pi\,a}{\sqrt{(a^2 - b^2)^3}}$

707 $\displaystyle\int_0^{2\pi} \frac{dx}{1 - 2\,a \cos x + a^2} = \frac{2\,\pi}{1 - a^2} \qquad 0 < a < 1$

708 $\displaystyle\int_0^{\pi} \frac{x \operatorname{sen} x\, dx}{1 - 2\,a \cos x + a^2} = \begin{cases} \frac{\pi}{a} \ln(1 + a); & \text{si } |a| < 1 \\ \pi \ln\left(1 + \frac{1}{a}\right); & |a| > 1 \end{cases}$

709 $\displaystyle\int_0^{\pi} \frac{\cos kx\, dx}{1 - 2\,a \cos x + a^2} = \frac{\pi\,a^k}{1 - a^2} \qquad \text{si } |a| < 1,\ k \in \mathbb{N}$

710 $\displaystyle\int_0^{\infty} \operatorname{sen} ax^2\, dx = \int_0^{\infty} \cos ax^2\, dx = \sqrt{\frac{\pi}{8\,a}}$

711 $\displaystyle\int_0^{\infty} \operatorname{sen} ax^n\, dx = \frac{\Gamma(1/n)}{n\,a^{1/n}} \operatorname{sen} \frac{\pi}{2\,n}; \qquad n > 1,\ \Gamma$: Función Gamma (Ver apéndice)

712 $\displaystyle\int_0^{\infty} \cos ax^n\, dx = \frac{\Gamma(1/n)}{n\,a^{1/n}} \cos \frac{\pi}{2\,n}; \qquad n > 1$

713 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} x}{\sqrt{x}}\, dx = \int_0^{\infty} \frac{\cos x}{\sqrt{x}}\, dx = \sqrt{\frac{\pi}{2}}$

714 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen} x}{x^p}\, dx = \frac{\pi}{2\,\Gamma(p) \operatorname{sen} \frac{p\,\pi}{2}}; \qquad 0 < p < 1$

715 $\displaystyle\int_0^{\infty} \frac{\cos x}{x^p}\, dx = \frac{\pi}{2\,\Gamma(p) \cos \frac{p\,\pi}{2}}; \qquad 0 < p < 1$

716 $\displaystyle\int_0^{\infty} \operatorname{sen} ax^2 \cos 2bx\, dx = \sqrt{\frac{\pi}{8\,a}} \left(\cos \frac{b^2}{a} - \operatorname{sen} \frac{b^2}{a} \right)$

717 $\displaystyle\int_0^{\infty} \cos ax^2 \operatorname{sen} 2bx\, dx = \sqrt{\frac{\pi}{8\,a}} \left(\cos \frac{b^2}{a} + \operatorname{sen} \frac{b^2}{a} \right)$

718 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen}^3 x}{x^3}\, dx = \frac{3\,\pi}{8}$

719 $\displaystyle\int_0^{\infty} \frac{\operatorname{sen}^4 x}{x^4}\, dx = \frac{\pi}{3}$

720 $\displaystyle\int_0^{\infty} \frac{\operatorname{tg} x}{x}\, dx = \frac{\pi}{2}$

721 $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \operatorname{tg}^n x} = \frac{\pi}{4}$

722 $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x\, dx}{\operatorname{sen} x} = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n - 1)^2}$

723 $\displaystyle\int_0^{1} \frac{\operatorname{tg}^{-1} x}{x}\, dx = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n - 1)^2}$

724 $\displaystyle\int_0^{1} \frac{\operatorname{sen}^{-1} x}{x}\, dx = \frac{\pi}{2} \ln 2$

725 $\displaystyle\int_0^1 \frac{1 - \cos x}{x}\, dx - \int_1^\infty \frac{\cos x}{x}\, dx = \gamma,$ γ: Constante de Euler

726 $\displaystyle\int_0^\infty \left(\frac{1}{1 + x^2} - \cos x \right) \frac{dx}{x} = \gamma$

727 $\displaystyle\int_0^\infty \frac{tg^{-1}\, px - tg^{-1}\, qx}{x}\, dx = \frac{\pi}{2} \ln \frac{p}{q}$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES RACIONALES E IRRACIONALES

728 $\displaystyle\int_0^\infty \frac{dx}{x^2 + a^2} = \frac{\pi}{2a}$

729 $\displaystyle\int_0^\infty \frac{x^{p-1}\, dx}{1 + x} = \frac{\pi}{\operatorname{sen} p\pi};$ $0 < p < 1$

730 $\displaystyle\int_0^\infty \frac{x^m\, dx}{x^n + a^n} = \frac{\pi\, a^{m-n+1}}{n \operatorname{sen}\left(\frac{m + 1}{n} \pi \right)};$ $0 < m + 1 < n$

731 $\displaystyle\int_0^\infty \frac{x^m\, dx}{1 + 2x \cos\beta + x^2} = \frac{\pi}{\operatorname{sen} m\pi} \cdot \frac{\operatorname{sen} m\beta}{\operatorname{sen}\beta}$

732 $\displaystyle\int_0^a \frac{dx}{\sqrt{a^2 - x^2}} = \frac{\pi}{2}$

733 $\displaystyle\int_0^a \sqrt{a^2 - x^2}\, dx = \frac{\pi\, a^2}{4}$

734 $\displaystyle\int_0^a x^m (a^n - x^n)^p\, dx = \frac{a^{m+np+1}}{n} \cdot \frac{\Gamma\left(\frac{m + 1}{n} \right) \cdot \Gamma(p + 1)}{\Gamma\left(\frac{m + 1}{n + p + 1} \right)}$ Γ: Función Gamma

735 $\displaystyle\int_0^\infty \frac{x^m\, dx}{(a^n + x^n)^r} = \frac{(-1)^{r-1}\, \pi\, a^{m-nr+1}}{n \operatorname{sen}\left(\frac{m + 1}{n} \pi \right) \cdot (r - 1)!} \cdot \frac{\Gamma\left(\frac{m + 1}{n} \right)}{\Gamma\left(\frac{m + 1}{n - r + 1} \right)};$ $0 < m + 1 < nr$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES EXPONENCIALES

736 $\displaystyle\int_0^\infty e^{-ax} \cos bx\, dx = \frac{a}{a^2 + b^2}$

737 $\displaystyle\int_0^\infty e^{-ax} \operatorname{sen} bx\, dx = \frac{b}{a^2 + b^2}$

738 $\displaystyle\int_0^\infty \frac{e^{-ax} \operatorname{sen} bx}{x}\, dx = tg^{-1} \frac{b}{a}$

739 $\displaystyle\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x}\, dx = \ln \frac{b}{a}$

740 $\displaystyle\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$

741 $\displaystyle\int_0^\infty e^{-ax^2} \cos bx\, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}\, e^{-(b^2/4a)}$

742 $\displaystyle\int_0^\infty e^{-(ax^2+bx+c)}\, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}\, e^{(b^2-4a)/4a} \cdot f_{cer} \frac{b}{2\sqrt{a}}$ Siendo $\displaystyle f_{cer}(p) = \frac{2}{\sqrt{\pi}} \int_p^\infty e^{-k^2}\, dk$

743 $\displaystyle\int_0^\infty e^{-(ax^2+bx+a)}\, dx = \sqrt{\frac{\pi}{a}}\, e^{(b^2-4a)/4a}$

744 $\displaystyle\int_0^\infty x^n e^{-ax}\, dx = \frac{\Gamma(n + 1)}{a^{n+1}}$ Γ: Función Gamma

745 $\displaystyle\int_0^\infty x^m e^{-ax^2}\, dx = \frac{\Gamma\left(\frac{m + 1}{2} \right)}{2\, a^{\frac{m+1}{2}}}$

746 $\displaystyle\int_0^\infty e^{-(ax^2+b/x^2)}\, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}\, e^{-2\sqrt{ab}}$

747 $\displaystyle\int_0^\infty \frac{x\, dx}{e^x - 1} = \sum_{n=1}^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$

748 $\displaystyle\int_0^\infty \frac{x^{n-1}\, dx}{e^x - 1} = \Gamma(n + 1) \sum_{k=1}^\infty \frac{1}{k^n};$ Γ: Función Gamma

Si n es par esta serie se puede hallar con ayuda de los números de Bernoulli (ver apéndice).

749 $\displaystyle\int_0^\infty \frac{x\, dx}{e^x + 1} = \sum_{n=1}^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

750 $\displaystyle\int_0^\infty \frac{x^{n-1}\, dx}{e^x + 1} = \Gamma(n + 1) \sum_{k=1}^\infty \frac{(-1)^{k+1}}{k^n}$ Γ: Función Gamma

751 $\displaystyle\int_0^\infty \frac{\operatorname{sen} mx\, dx}{e^{2\pi x} - 1} = \frac{1}{4} \operatorname{cotgh} \frac{m}{2} - \frac{1}{2m}$

752 $\displaystyle\int_0^\infty \left(\frac{1}{1 + x} - e^{-x} \right) \frac{dx}{x} = \gamma$ γ: Constante de Euler

753 $\displaystyle\int_0^\infty \frac{e^{-x^2} - e^{-x}}{x}\, dx = \frac{\gamma}{2}$

754 $\int_{0}^{\infty} \left(\frac{1}{e^{-x} - 1} - \frac{e^{-x}}{x} \right) dx = \gamma$ $\quad \gamma$: Constante de Euler

755 $\int_{0}^{\infty} \frac{e^{-ax} - e^{-bx}}{x \sec px}\, dx = \frac{1}{2} \ln \frac{b^2 + p^2}{a^2 + p^2}$

756 $\int_{0}^{\infty} \frac{e^{-ax} - e^{-bx}}{x \operatorname{cosec} px}\, dx = tg^{-1} \frac{b}{p} - tg^{-1} \frac{a}{p}$

757 $\int_{0}^{\infty} \frac{e^{-ax} (1 - \cos x)}{x^2}\, dx = \operatorname{cotg}^{-1} a - \frac{a}{2} \ln (a^2 + 1)$

INTEGRALES DEFINIDAS O IMPROPIAS DE FUNCIONES LOGARITMICAS

758 $\int_{0}^{1} x^m \ln^n x\, dx = \frac{(-1)^n\, n!}{(m + 1)^{n+1}}; \quad m > -1,\ n \in \mathbb{N}_0$

759 $\int_{0}^{1} x^m \ln^n x\, dx = \frac{(-1)^n\, \Gamma(n + 1)}{(m + 1)^{n+1}}; \quad m > -1,\ n \notin \mathbb{N}_0,\ \Gamma$: Función Gamma

760 $\int_{0}^{1} \frac{\ln x}{1 + x}\, dx = -\frac{\pi^2}{12}$

761 $\int_{0}^{1} \frac{\ln x}{1 - x}\, dx = -\frac{\pi^2}{6}$

762 $\int_{0}^{1} \frac{\ln (1 + x)}{x}\, dx = \frac{\pi^2}{12}$

763 $\int_{0}^{1} \frac{\ln (1 - x)}{x}\, dx = -\frac{\pi^2}{6}$

764 $\int_{0}^{1} \ln x \ln (1 + x)\, dx = 2\,(1 - \ln 2) - \frac{\pi^2}{12}$

765 $\int_{0}^{1} \ln x \ln (1 - x)\, dx = 2 - \frac{\pi^2}{6}$

766 $\int_{0}^{\infty} \frac{x^{p-1} \ln x}{1 + x}\, dx = -\pi^2 \operatorname{cosec} p\pi \cdot \operatorname{cotg} p\pi \quad 0 < p < 1$

767 $\int_{0}^{1} \frac{x^m - x^n}{\ln x}\, dx = \ln \frac{m + 1}{n + 1}$

768 $\int_{0}^{\infty} e^{-x} \ln x\, dx = -\gamma$ $\quad \gamma$: Constante de Euler

769 $\int_{0}^{\infty} e^{-x^2} \ln x\, dx = -\frac{\sqrt{\pi}}{4} (\gamma + 2 \ln 2)$

770 $\int_{0}^{\infty} \ln \frac{e^x + 1}{e^x - 1}\, dx = \frac{\pi^2}{4}$

771 $\int_{0}^{\frac{\pi}{2}} \ln \operatorname{sen} x\, dx = \int_{0}^{\frac{\pi}{2}} \ln \cos x\, dx = -\frac{\pi \ln 2}{2}$

772 $\int_{0}^{\frac{\pi}{2}} (\ln \operatorname{sen} x)^2\, dx = \int_{0}^{\frac{\pi}{2}} (\ln \cos x)^2\, dx = \frac{\pi^3}{4!} + \frac{\pi \ln^2 2}{2}$

773 $\int_{0}^{\pi} x \ln \operatorname{sen} x\, dx = -\frac{\pi^2 \ln 2}{2}$

774 $\int_{0}^{\frac{\pi}{2}} \operatorname{sen} x \ln \operatorname{sen} x\, dx = \ln \frac{2}{e}$

775 $\int_{0}^{2\pi} \ln (a + b \operatorname{sen} x)\, dx = \int_{0}^{2\pi} \ln (a + b \cos x)\, dx = 2\pi \ln \left(a + \sqrt{a^2 - b^2} \right)$

776 $\int_{0}^{\pi} \ln (a + b \cos x)\, dx = \pi \ln \frac{a + \sqrt{a^2 - b^2}}{2}$

777 $\int_{0}^{\pi} \ln (a^2 + 2\, a\, b \cos x + b^2)\, dx = \begin{cases} 2\pi \ln b; & \text{si } 0 < a \leq b \\ 2\pi \ln a; & \text{si } 0 < b \leq a \end{cases}$

778 $\int_{0}^{\frac{\pi}{4}} \ln (1 + tg\, x)\, dx = \frac{\pi \ln 2}{8}$

779 $\int_{0}^{\frac{\pi}{2}} \sec x \ln \left(\frac{1 + b \cos x}{1 + a \cos x} \right) dx = \frac{(\cos^{-1} a)^2 - (\cos^{-1} b)^2}{2}$

780 $\int_{0}^{a} \ln \left(2 \operatorname{sen} \frac{\pi}{2} \right) dx = -\sum_{n=1}^{\infty} \frac{\operatorname{sen} an}{n^2}$

INTEGRALES IMPROPIAS DE FUNCIONES HIPERBOLICAS

781 $\int_{0}^{\infty} \frac{\operatorname{sen} ax}{\operatorname{senh} bx}\, dx = \frac{\pi}{2b}\, tgh \frac{a\pi}{2b}$

782 $$\int_0^\infty \frac{\cos ax}{\cosh bx}\,dx = \frac{\pi}{2\,b}\,\operatorname{sech}\frac{a\,\pi}{2\,b}$$

783 $$\int_0^\infty \frac{x\,dx}{\operatorname{senh} ax} = \frac{\pi^2}{4\,a^2}$$

784 $$\int_0^\infty \frac{x^n\,dx}{\operatorname{senh} ax} = \frac{2^{n+1}-1}{2^{n+1}\cdot a^{n+1}}\,\Gamma(n+1)\sum_{k=1}^{\infty}\frac{1}{k^{n+1}};\qquad \Gamma\text{: Función Gamma}$$

785 $$\int_0^\infty \frac{\operatorname{senh} ax}{e^{bx}+1}\,dx = \frac{\pi}{2\,b}\operatorname{cosec}\frac{a\,\pi}{b} - \frac{1}{2\,a}$$

786 $$\int_0^\infty \frac{\operatorname{senh} ax}{e^{bx}-1}\,dx = \frac{1}{2\,a} - \frac{\pi}{2\,b}\operatorname{cotg}\frac{a\,\pi}{b}$$

APENDICE

FUNCION GAMMA

Definición: $$\Gamma(n) = \int_0^\infty t^{n-1}\,e^{-t}\,dt; \qquad n > 0$$

Fórmula de recurrencia: $\Gamma(n+1) = n\,\Gamma(n)$

Si $n \in \mathbb{N} \Rightarrow \Gamma(n+1) = n!$ Si $n < 0 \Rightarrow \Gamma(n) = \dfrac{\Gamma(n+1)}{n}$

Propiedades: a) $\Gamma(p)\,\Gamma(1-p) = \dfrac{\pi}{\operatorname{sen} p\pi}$ b) $\dfrac{2^{2x-1}}{\sqrt{\pi}} = \dfrac{\Gamma(2\,x)}{\Gamma(x)\,\Gamma\left(x+\frac{1}{2}\right)}$

FUNCION BETA

Definición: $$B(m,n) = \int_0^1 t^{m-1}\,(1-t)^{n-1}\,dt; \qquad m > 0;\ n > 0$$

Relación con la función Gamma: $B(m,n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$

Otras formas de expresar la función Beta: $B(m,n) = B(n,m) =$

$$= 2\int_0^{\frac{\pi}{2}} \operatorname{sen}^{2m-1} x\,\cos^{2n-1} x\,dx = \int_0^\infty u^{m-1}\,(1+u)^{-m-n}\,du = r^n\,(r+1)^m \int_0^1 \frac{t^{m-1}\,(1-t)^{n-1}}{(r+t)^{m+n}}\,dt$$

NUMEROS DE BERNOULLI Y EULER

a) Bernoulli: los números B_1; B_2; B_3; ··· se definen por las series:

RAB 16

$$\frac{x}{e^x-1} = 1 - \frac{x}{2} + \frac{B_1\,x^2}{2!} + \frac{B_2\,x^4}{4!} + \frac{B_3\,x^6}{6!} + \cdots \qquad |x| < 2\,\pi$$

o también $$1 - \frac{x}{2}\operatorname{cotg}\left(\frac{x}{2}\right) = \frac{B_1\,x^2}{2!} + \frac{B_2\,x^4}{4!} + \frac{B_3\,x^6}{6!} + \cdots \qquad |x| < \pi$$

b) Euler: los números de Euler E_1; E_2; E_3; ··· se definen por las series:

$$\operatorname{sech} x = 1 - \frac{E_1\,x^2}{2!} + \frac{E_2\,x^4}{4!} - \frac{E_3\,x^6}{6!} + \cdots \qquad |x| < \pi/2$$

$$\sec x = 1 + \frac{E_1\,x^2}{2!} + \frac{E_2\,x^4}{4!} + \frac{E_3\,x^6}{6!} + \cdots \qquad |x| < \pi/2$$

Tabla de algunos números B_n y E_n

n	B_n	E_n
1	1/6	1
2	1/30	5
3	1/42	61
4	1/30	1385
5	5/66	50521
6	691/2730	2702765
7	7/6	199360981
8	3617/510	19391512145
9	43867/798	2404879675441
10	174611/330	370371188237525

DADA $ax^2 + bx + c = 0$

$x_1 + x_0 = -\dfrac{b}{a}$

$x_1 \cdot x_2 = \dfrac{c}{a}$

IC → DESCRIBE UNA CURVA C regular $\in D \subset \mathbb{R}^3$

Definir Inter c/o lcyo b C = $\overline{AB}$

Identidades importantes.

$$\operatorname{sh} x = \frac{e^{x} - e^{-x}}{2}$$

$$\operatorname{ch} x = \frac{e^{x} + e^{-x}}{2}$$

$$\operatorname{sen} x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\operatorname{sen} z = \operatorname{sh} y \cos x + i \operatorname{ch} y \operatorname{sen} x$$

$z \in \mathbb{C}$

$$e^{z} = \sum_{n=0}^{\infty} \frac{z^n}{n!} \qquad |z| < \infty$$

$$\operatorname{sen} z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1} \qquad |z| < \infty$$

$$\cos z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n} \qquad |z| < \infty$$

$$\operatorname{sh} z = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!} \qquad |z| < \infty$$

$$\operatorname{ch} z = \sum_{n=0}^{\infty} \frac{z^{2n}}{2n!} \qquad |z| < \infty$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n \qquad |z| < 1$$

$$\frac{1}{1+z} = \sum_{n=0}^{\infty} (-1)^n z^n \qquad |z| < 1$$

$$\frac{1}{1-z^2} = \sum_{n=0}^{\infty} z^{2n} \qquad |z| < 1$$

$$\ln(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} z^n \qquad |z| < 1$$

$$(1+z)^m = 1 + \sum_{n=1}^{+\infty} \frac{m(m-1)(m-2)\ldots(m-n+1)}{n!} x^n \qquad |z| < 1$$

$m \in \mathbb{C}$